Day 314 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Exponential equations reducing to quadratics (All boards)

Solve $5^{2x} - 425^x + 425 = 0$.
Give exact roots, or 3 s.f. where the answer is not exact.

Let $u = 5^x$: $u^2 - 42u + 425 = 0$, so $(u - 25)(u - 17) = 0$

$5^x = 25$ gives $x = 2$; $5^x = 17$ gives $x = \frac{\ln 17}{\ln 5} = 1.76$

Answer: x = 2 or x = 1.76

2 Circle: centre, radius and position of a point (AQA, Edexcel, OCR, MEI, Eduqas)

A circle has equation $x^2 + y^2 - 6x - 12y + 41 = 0$.
(a) Find the centre and radius of the circle.
(b) Determine whether P(1, 2) lies inside or outside the circle.

Complete the square: $(x - 3)^2 + (y - 6)^2 = 4$
Centre (3, 6), radius 2
Distance^2 from centre to P = $(-2)^2 + (-4)^2 = 20$
Compare with $r^2 = 4$
Answer: (a) centre (3, 6), radius 2 (b) P is outside the circle

3 Projectile: perpendicular directions (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from the ground at 22 m s^{-1} at angle α above the horizontal, $\tan\alpha = \frac{12}{5}$. At time T s, while in flight, it is moving perpendicular to its initial direction. Find (a) T, (b) the height of the ball then. Take g = 9.8. Give answers to 3 s.f. if not exact.

Initial velocity $(\frac{110}{13}, \frac{264}{13})$. At time T the velocity is $(\frac{110}{13}, \frac{264}{13} - 9.8T)$.

Perpendicular: scalar product = 0: $(\frac{110}{13})^2 + \frac{264}{13}(\frac{264}{13} - 9.8T) = 0$

$22^2 - 9.8T \times \frac{264}{13} = 0$ (using $\sin^2\alpha + \cos^2\alpha = 1$)

(a) $T = 22 \div (9.8 \times \frac{12}{13}) = \frac{715}{294}$

(b) $h = \frac{264}{13} \times \frac{715}{294} - 4.9 \times \frac{715}{294}^2 = \frac{10285}{504}$

Answer: (a) T = 2.43 s (b) 20.4 m

4 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

For a sample of 10 pairs of values of time spent on social media and exam score, r = −0.751.
The critical value for a two-tailed test at the 5% level is 0.6319.
Test, at this level, whether there is any correlation between the two variables.

H_0: $\rho = 0$, H_1: $\rho \neq 0$
Critical value 0.6319
$|r| = 0.751 > 0.6319$
This is in the critical region, so reject H_0.
There is evidence of negative correlation.
Answer: Reject H_0: evidence of correlation

5 Double angle equations (All boards)

Solve $\cos 2x - 4\sin x = -5$
for $0° \leq x \leq 360°$.

$\cos 2x \equiv 1 - 2\sin^2 x$: $2\sin^2 x + 4\sin x - 6 = 0$
Solve the quadratic: $\sin x = 1$ or $\sin x = -3$
$\sin x = 1$: $x = 90°$
$\sin x = -3$ is impossible
Answer: x = 90°

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6 Three-set Venn diagram counts (All boards)

Of 44 students, 21 study Maths (M), 20 Physics (P) and 21 Chemistry (C).
7 study M and P, 11 study M and C, 8 study P and C, and 5 study all three.
One student is chosen at random. Given that a student studies exactly two subjects, find the probability that they study Physics.

Venn regions: M only 8, P only 10, C only 7,
MP only 2, MC only 6, PC only 3, all three 5, outside all 3
Exactly two subjects: 2 + 6 + 3 = 11
Of these, Physics: 2 + 3 = 5

Probability = $\frac{5}{11}$

Answer: $\frac{5}{11}$

7 Recurrence relations: unknown first term (All boards)

A sequence is defined by $u_1 = k$ and $u_{n+1} = 2u_n + 2$.

Given that $u_3 = -6$, find (a) the value of k, (b) $\sum_{r=1}^{4} u_r$.

$u_2 = 2k + 2$, $u_3 = 2(2k + 2) + 2$
$4k + 6 = -6$ so $k = -3$
Terms: $-3, -4, -6, -10$
Sum = -23
Answer: (a) k = −3 (b) −23

8 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $3x^2 + 2xy + 3y^2 = 24$.
Find the coordinates of the points where the tangent is parallel to the x-axis.

$\frac{dy}{dx} = -\frac{6x + 2y}{2x + 6y}$
Horizontal tangent: $6x + 2y = 0$, so $y = -3x$
Substitute: $24x^2 = 24$, so $x^2 = 1$ and $x = \pm 1$
Points: $(1, -3)$ and $(-1, 3)$
Answer: (1, −3) and (−1, 3)

9 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 2 kg particle hangs in equilibrium from two light strings, as shown. Find the tension in each string (T_1 in the string at angle α). Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = \frac{5}{13}$, $\cos\alpha = \frac{12}{13}$, $\sin\beta = 0.8$, $\cos\beta = 0.6$

Horizontal: $T_1 \cos\alpha = T_2 \cos\beta$, so $T_2 = \frac{20}{13} T_1$

Vertical: $T_1 \sin\alpha + T_2 \sin\beta = 2g = 19.6$

$T_1(\frac{5}{13} + \frac{16}{13}) = 19.6$

$T_1 = 12.1$, $T_2 = 18.7$

Answer: T_1 = 12.1 N, T_2 = 18.7 N

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10 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{x + 1}{(1 + 3x)(1 + 2x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) up to x^2.

(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 + 3x)} + \frac{B}{(1 + 2x)}$: A = 2, B = −1

(b) $2(1 + 3x)^{-1} = 2(1 - 3x + 9x^2 ...)$

Add the series for both fractions.

(c) Need |3x| < 1 and |2x| < 1

Answer: (a) $\frac{2}{(1 + 3x)} + \frac{-1}{(1 + 2x)}$ **(b)** $1 - 4x + 14x^2$ **(c)** $|x| < \frac{1}{3}$