Day 313 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A systematic sample is taken from a numbered list of 420 items.
The 4th item chosen is number 43 and the 8th item chosen is number 91.
Find the sampling interval, the number of the first item chosen and the size of the sample.

Interval = $(91 - 43) \div (8 - 4) = 12$
First item = $43 - 3 \times 12 = 7$
Sample size = $420 \div 12 = 35$
Answer: Interval 12, first 7, size 35

2 Car and trailer (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 1500 kg car tows a 200 kg trailer by a light tow-bar on a horizontal road. Driving force 1250 N; resistances 300 N (car), 100 N (trailer). Find (a) the acceleration and the tow-bar tension. At 8 m s^{-1} the tow-bar breaks. (b) How far does the trailer then travel?

Whole system: $1250 - 300 - 100 = 1700\,a$, so $a = 0.5$
Trailer: $T - 100 = 200\,a$, so $T = 100 + 200 \times 0.5 = 200$
(b) After the break the trailer decelerates at $100 \div 200 = 0.5$ m s^{-2}
$0 = 8^2 - 2 \times 0.5 \times s$, so $s = 64$
Answer: (a) 0.5 m s^{-2}, 200 N (b) 64 m

3 Addition formulae with given sin and cos (All boards)

A is acute and B is obtuse, with $\sin A = \frac{8}{17}$ and $\cos B = -\frac{3}{5}$.
Find the exact values of cos(A+B) and sin(A+B).

$\cos A = \frac{15}{17}$ (A acute), $\sin B = \frac{4}{5}$ (B obtuse), so $\tan A = \frac{8}{15}$, $\tan B = -\frac{4}{3}$

$\cos(A+B) = \cos A \cos B - \sin A \sin B = \frac{15}{17} \times \left(-\frac{3}{5}\right) - \frac{8}{17} \times \frac{4}{5} = -\frac{77}{85}$

$\sin(A+B) = \sin A \cos B + \cos A \sin B = \frac{8}{17} \times \left(-\frac{3}{5}\right) + \frac{15}{17} \times \frac{4}{5} = \frac{36}{85}$

Answer: $\cos(A+B) = -\frac{77}{85}$, $\sin(A+B) = \frac{36}{85}$

4 Circle with a given diameter (AQA, Edexcel, OCR, MEI, Eduqas)

A(−1, −3) and B(−7, 5) are the ends of a diameter of a circle.
(a) Find the equation of the circle, in the form $x^2 + y^2 + ax + by + c = 0$.
(b) Show that C(0, 4) lies on the circle.

Centre = midpoint of AB = (−4, 1)
$r^2 = (3)^2 + (-4)^2 = 25$
$(x + 4)^2 + (y - 1)^2 = 25$: $x^2 + y^2 + 8x - 2y - 8 = 0$
(b) $CM^2 = (4)^2 + (3)^2 = 25 = r^2$
Answer: (a) $x^2 + y^2 + 8x - 2y - 8 = 0$

5 Regression line from summary data (AQA, Edexcel, OCR, MEI, Eduqas)

n = 8 pairs of (x, y): $\Sigma x = 177$, $\Sigma y = 446$, $\Sigma x^2 = 4231$, $\Sigma xy = 10328$.
(a) Find the equation of the regression line of y on x (3 s.f.).
(b) Estimate y when x = 25.

$S_{xx} = 4231 - \frac{177^2}{8} = 315$, $S_{xy} = 10328 - \frac{177 \times 446}{8} = 460$

$b = \frac{S_{xy}}{S_{xx}} = 1.46$
$a = \bar{y} - b\,\bar{x} = 55.8 - (1.46)(22.1) = 23.4$
(b) $y = 23.4 + (1.46)(25) = 60.0$
Answer: (a) $y = 23.4 + 1.46x$ (b) 60.0

6 Smooth incline: passing a point twice (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is projected up a line of greatest slope of a smooth plane inclined at α. It passes A, 17.15 m from the start, after 2 s and again after 7 s. Find (a) the speed of projection, (b) α to 1 d.p. Take g = 9.8. Give answers to 3 s.f. if not exact.

Deceleration a = g sin α (unknown). $s = ut - \frac{1}{2}at^2$ at both times:
$u\times2 - \frac{1}{2}a\times2^2 = u\times7 - \frac{1}{2}a\times7^2 = 17.15$
Subtracting: $u(7 - 2) = \frac{1}{2}a(49 - 4)$, so $u = \frac{1}{2}a(9)$
Substituting: $\frac{1}{2}a(9)(2) - \frac{1}{2}a\times2^2 = 17.15$, so $\frac{1}{2}a\times14 = 17.15$, $a = 2.45$
(a) $u = 1.225\times9 = 11.025$
(b) $9.8 \sin\alpha = 2.45$, $\sin\alpha = 0.25$, $\alpha = 14.5°$

Answer: (a) 11.0 m s^{-1} (b) 14.5°

7 Binomial expansion: terms with negative powers of x (All boards)

Find the coefficient of x^3 in the expansion of

$(2x^2 + \frac{2}{x})^9$.

General term: ${}^9C_r\,(2x^2)^{9-r}\,(2x^{-1})^r$
Power of x: $2(9 - r) - 1r = 3$
$18 - 3r = 3$ so $r = 5$
Coefficient = ${}^9C_5 \times 2^4 \times 2^5 = 64512$

Answer: 64512

8 Double and triple angle values (All boards)

θ is obtuse and $\sin\theta = \frac{15}{17}$.

Find the exact values of sin 2θ, cos 2θ and tan 2θ.

$\cos\theta = -\sqrt{1 - \left(\frac{15}{17}\right)^2} = -\frac{8}{17}$ (obtuse)

$\sin2\theta = 2\sin\theta\cos\theta = 2 \times \frac{15}{17} \times (-\frac{8}{17}) = -\frac{240}{289}$

$\cos2\theta = 1 - 2\sin^2\theta = 1 - 2 \times \left(\frac{15}{17}\right)^2 = -\frac{161}{289}$

$\tan2\theta = \frac{\sin2\theta}{\cos2\theta} = \frac{240}{161}$

Answer: $\sin2\theta = -\frac{240}{289}$, $\cos2\theta = -\frac{161}{289}$, $\tan2\theta = \frac{240}{161}$

9 Exact values from the addition formulae (All boards)

By writing 15° as 45° − 30°, find the exact value of

cos 15°, in the form $\frac{\sqrt{a} \pm \sqrt{b}}{c}$.

$\cos15° = \cos(45° - 30°) = \cos45°\cos30° + \sin45°\sin30°$
$= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \times \frac{1}{2}$
$= \frac{\sqrt{2} + \sqrt{6}}{4}$

Answer: $\frac{\sqrt{2} + \sqrt{6}}{4}$

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10 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{7x - 3}{(1 - x)(1 + x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) up to x^2.

(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 - x)} + \frac{B}{(1 + x)}$: A = 2, B = −5

(b) $2(1 - x)^{-1} = 2(1 + x + 1x^2 ...)$

Add the series for both fractions.

(c) Need |x| < 1 and |x| < 1

Answer: (a) $\frac{2}{(1 - x)} + \frac{-5}{(1 + x)}$ **(b)** $-3 + 7x - 3x^2$ **(c)** $|x| < 1$