Day 312 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Binomial expansion: terms with negative powers of x (All boards)

Find the coefficient of x^5 in the expansion of $(x^2 + \frac{1}{x})^{10}$.

General term: ${}^{10}C_r\,(x^2)^{10-r}\,(x^{-1})^r$
Power of x: $2(10 - r) - 1r = 5$
$20 - 3r = 5$ so $r = 5$
Coefficient = ${}^{10}C_5 \times 1^5 \times 1^5 = 252$
Answer: 252

2 Critical values for a binomial distribution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(15, 0.3)$.
Find the largest value of c such that $P(X \leq c) < 0.05$.

$P(X \leq 1) = 0.0353$
$P(X \leq 2) = 0.127$
$0.0353 < 0.05$ but $0.127 > 0.05$
Answer: c = 1

3 Stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A driver at constant speed sees a hazard 91.2 m ahead. After a reaction time of 0.8 s she brakes with deceleration 4 m s^{-2} and stops exactly at the hazard. Find (a) her initial speed, (b) the total time from seeing the hazard to stopping.

Let the initial speed be v. Reaction distance = 0.8 v, braking distance = $v^2 \div (2\times4)$
$0.8\,v + v^2 \div 8 = 91.2$, so $v^2 + 6.4\,v - 729.6 = 0$
$(v - 24)(v + 30.4) = 0$, so $v = 24$
(b) Braking time = $24 \div 4 = 6$ s; total = 0.8 + 6
Answer: (a) 24 m s^{-1} (b) 6.8 s

4 Fixed point iteration (All boards)

The equation $x^2 = x + 7$ has a root α. Use the iteration $x_{n+1} = \sqrt{x + 7}$ with $x_0 = 3$ to find x_1, x_2 and x_3 to 4 d.p., and hence the value of α to 2 d.p. (continue until the 2 d.p. value is settled).

$x_1 = \sqrt{3 + 7} = 3.1623$
$x_2 = 3.1878$, $x_3 = 3.1918$
Continuing, the values settle at 3.1926, so α = 3.19
Answer: x_1 = 3.1623, x_2 = 3.1878, x_3 = 3.1918; α = 3.19

5 Discriminant: range of k (All boards)

The equation $x^2 + (2k)x + (2k) = 0$ has no real roots.
Find the set of possible values of k.

Discriminant: $b^2 - 4ac = (2k)^2 - 4(2k)$
$= 4k^2 - 8k$
Roots of the quadratic in k: k = 0 and k = 2
Required: $b^2 - 4ac < 0$
Answer: 0 < k < 2

6 Exponential growth and decay modelling (All boards)

The number of fish in a lake is modelled by $N = 2000\,e^{kt}$, where t is time in years.
After 4 years, N = 4000.
Find (a) the exact value of k, (b) the time for N to reach 100000, to 3 s.f.

$2000\,e^{4k} = 4000$, so $e^{4k} = 2$ and $k = \frac{\ln 2}{4}$

$k = 0.173$ (3 s.f.)

$2000\,e^{kt} = 100000$ gives $kt = \ln 50$, so $t = \frac{\ln 50}{k} = 22.6$ years

Answer: (a) $k = \frac{\ln 2}{4}$ (b) t = 22.6 years

7 Binomial expansion: product with a bracket (All boards)

Find the coefficient of x^3 in the expansion of
$(1 - 2x)(2 - x)^5$.

In $(2 - x)^5$: coefficient of x^2 is 80, of x^3 is -40
Coefficient = $1 \times (-40) + (-2) \times 80$
Answer: -200

8 Type I and Type II errors (AQA, Edexcel, OCR, MEI, Eduqas)

$X \sim B(24, p)$. H_0: $p = 0.3$ is tested against H_1: $p < 0.3$ using a 5% significance level.
(a) Find the critical region. (b) State the probability of a Type I error.
(c) Find the probability of a Type II error when $p = 0.1$ (3 s.f.).

(a) Largest/smallest region with probability ≤ 0.05 under H_0: $X \leq 3$
(b) Type I error = P(reject H_0 | $p = 0.3$) = 0.0424
(c) Type II error = P(not in critical region | $p = 0.1$)
$P(X \geq 4)$ when $p = 0.1$: 0.214
Answer: (a) $X \leq 3$ (b) 0.0424 (c) 0.214

9 Implicit differentiation: products and cubes (All boards)

A curve has equation $2x^2y + y^3 + 3x = 15$.
Find $\frac{dy}{dx}$ in terms of x and y, and the gradient of the normal at (1, 2).

Differentiate: $4xy + 2x^2\frac{dy}{dx} + 3y^2\frac{dy}{dx} + 3 = 0$

$\frac{dy}{dx} = -\frac{4xy + 3}{2x^2 + 3y^2}$

At (1, 2): $\frac{dy}{dx} = -\frac{11}{14}$, so the normal has gradient $\frac{14}{11}$

Answer: $\frac{dy}{dx} = -\frac{4xy + 3}{2x^2 + 3y^2}$; normal gradient $\frac{14}{11}$

10 Range of horizontal force on a rough slope (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 4 kg particle is in equilibrium on a rough plane inclined at α, $\tan\alpha = \frac{3}{4}$, $\mu = 0.2$. A horizontal force of P N, in the vertical plane through a line of greatest slope, pushes it towards the plane. Find the least and greatest values of P. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = 0.6$, $\cos\alpha = 0.8$, weight W = 39.2 N. Normal reaction $R = W\cos\alpha + P\sin\alpha$
Least P: about to slide down, friction $F = \mu R$ up the slope
$P\cos\alpha + \mu(W\cos\alpha + P\sin\alpha) = W\sin\alpha \rightarrow P = W(\sin\alpha - \mu\cos\alpha) \div (\cos\alpha + \mu\sin\alpha) = 18.7$
Greatest P: about to slide up, friction down the slope
$P\cos\alpha = W\sin\alpha + \mu(W\cos\alpha + P\sin\alpha) \rightarrow P = W(\sin\alpha + \mu\cos\alpha) \div (\cos\alpha - \mu\sin\alpha) = 43.8$
Answer: least 18.7 N, greatest 43.8 N