Day 311 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Algebraic fractions (All boards)

Express $\frac{1}{(x-2)} - \frac{5}{(x+1)}$ as a single fraction in its simplest form.

Common denominator $(x-2)(x+1)$

Numerator: $1(x+1) - 5(x-2) = -4x + 11$

Answer: $\mathbf{\frac{-4x+11}{(x-2)(x+1)}}$

2 Area between curves and the x-axis (with sketch) (All boards)

The diagram shows the curve C: $y = x(4-x)$ and the line L: $y = 2x$.

Find the area of the shaded region between C and L.

Intersect: $x(4-x) = 2x$ gives $x = 0$ or $x = 2$

Area $= \int(C - L)\,dx = \int_0^2 (2x - x^2)\,dx = [\frac{2x^2}{2} - \frac{x^3}{3}]$

$= \frac{2^3}{2} - \frac{2^3}{3} = \frac{4}{3}$

Answer: $\mathbf{\frac{4}{3}}$

3 Coding mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 20 values of x, $\Sigma(x-50) = 40$ and $\Sigma(x-50)^2 = 85$.

Find the mean and standard deviation of x.

Let $y = x - 50$: $\bar{y} = \frac{40}{20} = 2$

Mean of $x = 2 + 50 = 52$

Variance $= \frac{85}{20} - 2^2 = 0.25$

Subtracting a constant does not change spread, so sd = 0.5

Answer: mean 52, sd 0.5

4 Smallest whole number solving an exponential inequality (All boards)

£20000 is invested at 8% compound interest per year.

Find the smallest whole number of years for the value to exceed £40000.

$20000 \times 1.08^n > 40000$, so $1.08^n > 2$

$n \ln 1.08 > \ln 2$, so $n > \frac{\ln 2}{\ln 1.08} = 9.01$

$n = 10$

Answer: n = 10 years

5 Transformations of a turning point (All boards)

The curve $y = f(x)$ has a minimum point at $(-1, 6)$.

Find the coordinates of the corresponding point on:

(a) $y = f(x-1)$

(b) $y = f(-x)$

(c) $y = f(\frac{x}{2})$

(a) $y = f(x-1)$: $(0, 6)$

(b) $y = f(-x)$: $(1, 6)$

(c) $y = f(\frac{x}{2})$: $(-2, 6)$

A stretch or translation moves the point in the stated direction only.

Answer: (a) (0, 6); (b) (1, 6); (c) (−2, 6)

6 Smallest number of trials (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a seed germinates is 0.02 on each independent trial.
Find the least number of trials needed so that the probability of this happening at least once is greater than 0.995.

P(at least one) = $1 - 0.98^n > 0.995$

$0.98^n < 0.005$

$n > \frac{\log 0.005}{\log 0.98} = 262$

Least integer n = 263

Answer: 263

7 Particles on two smooth inclined planes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The system is released from rest. Find (a) the magnitude of the acceleration and which particle moves down, (b) the tension. Take g = 9.8. Give answers to 3 s.f. if not exact.

A: weight component along plane = 8 g sin 30°; B: 3 g sin 45°

A moves down, B up (8 sin 30° > 3 sin 45°): F = ma for the system

8 g sin 30° − 3 g sin 45° = (8 + 3)a

(a) a = 1.67

(b) For A: 8 g sin 30° − T = 8 a, so T = 25.8

Answer: (a) 1.67 m s^{-2}, A moves down (b) 25.8 N

8 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{6} - 3) = -5 - 3\sqrt{6}$, writing x in the form $p + q\sqrt{6}$.

$x = \frac{-5 - 3\sqrt{6}}{\sqrt{6} - 3}$

Multiply top and bottom by $\sqrt{6} + 3$.

Numerator: $(-5 - 3\sqrt{6})(\sqrt{6} + 3) = -33 - 14\sqrt{6}$

Denominator: $6 - 9 = -3$

Answer: $x = 11 + \frac{14}{3}\sqrt{6}$

9 Closest approach of two particles (Edexcel, OCR, MEI)

At t = 0, P and Q have position vectors $(-12\mathbf{i} - \mathbf{j})$ m and $(-5\mathbf{i} - 2\mathbf{j})$ m and velocities $(4\mathbf{i})$ m s^{-1} and $(-3\mathbf{j})$ m s^{-1}. Find (a) the time when they are closest, (b) the least distance between them.

$PQ = r_P - r_Q = (-7\mathbf{i} + \mathbf{j}) + t(4\mathbf{i} + 3\mathbf{j})$

$d^2 = (4t - 7)^2 + (3t + 1)^2 = 25t^2 - 50t + 50$

(a) $d(d^2)/dt = 0$: 50 t + −50 = 0, so t = 1

(b) At t = 1: $PQ = -3\mathbf{i} + 4\mathbf{j}$, so d = √25

Answer: (a) t = 1 s (b) 5 m

10 Integration by parts (All boards)

Use integration by parts to find the exact value of

$\int_0^3 x\, e^{-3x}\, dx$.

$u = x$, $\frac{dv}{dx} = e^{-3x}$: $\int xe^{-3x}\, dx = [-\frac{xe^{-3x}}{3}] + \int \frac{e^{-3x}}{3}\, dx$

$= [-\frac{xe^{-3x}}{3} - \frac{e^{-3x}}{9}]$

$= \frac{1 - 10e^{-9}}{9}$

Answer: $\frac{1 - 10e^{-9}}{9}$