Day 310 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Equilibrium on a slope: range of force (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 3 kg particle rests on a rough plane inclined at α, $\tan\alpha = \frac{15}{8}$, $\mu = 0.25$. A force of P N acts up a line of greatest slope and the particle stays at rest. Find the range of values of P. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = \frac{15}{17}$, $\cos\alpha = \frac{8}{17}$, $R = 3g\cos\alpha = \frac{1176}{85}$ N

Limiting friction $= \mu R = \frac{294}{85}$ N

P smallest: friction acts up the slope: $P + \mu R = 3g\sin\alpha$, $P = \frac{441}{17} - \frac{294}{85}$

P greatest: friction acts down the slope: $P = \frac{441}{17} + \frac{294}{85}$

Answer: 22.5 N ≤ P ≤ 29.4 N

2 Exact definite integrals of standard functions (All boards)

Find the exact value of

$\int_0^2 (4x+3)^4\,dx$.

Integrate: $\left[\frac{(4x+3)^5}{20}\right]$

Substitute the limits and subtract: $\frac{40202}{5}$

Answer: $\frac{40202}{5}$

3 Correcting a recorded value (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

12 values have $\Sigma x = 480$ and $\Sigma x^2 = 19968$.
One value was recorded as 25 but should have been 46.
Find the correct mean and standard deviation (3 s.f.).

$\Sigma x = 480 - 25 + 46 = 501$

$\Sigma x^2 = 19968 - 25^2 + 46^2 = 21459$

Mean $= \frac{501}{12} = 41.8$

sd $= \sqrt{\frac{21459}{12} - (41.8)^2} = 6.72$

Answer: mean 41.8, sd 6.72

4 Circle and line: tangent condition (AQA, Edexcel, OCR, MEI, Eduqas)

The line $3x - 4y = k$ is a tangent to the circle
$(x-2)^2 + (y+4)^2 = 25$.
Find the two possible values of k.

Centre (2, −4), radius 5
Distance from centre to the line = radius:

$\frac{|22 - k|}{5} = 5$

$|22 - k| = 25$

Answer: k = −3 or k = 47

5 Symmetric interval about the mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the length (mm), is $N(81, 2^2)$.
Find the symmetric limits about the mean between which the central 90% of values lie (3 s.f.).

Each tail has area 0.05, so $P(Z < z) = 0.95$
$z = 1.64$
Limits: $81 \pm 1.64 \times 2$
= 77.7 and 84.3
Answer: 77.7 to 84.3

6 Area and perimeter of a segment (exact) (All boards)

The circle has centre O and radius 20 cm. Angle AOB = $\frac{3\pi}{4}$.

Find the exact area of the shaded segment.

Sector area = $\frac{1}{2}r^2\theta = 150\pi$; triangle OAB = $\frac{1}{2}r^2\sin\theta = 100\sqrt{2}$

Segment area = sector − triangle = $150\pi - 100\sqrt{2}$

Answer: area = $150\pi - 100\sqrt{2}$ cm^2

7 Partial fractions: improper fraction (All boards)

Express $\frac{2x^2 - x - 8}{(x+1)(x-4)}$ in the form $2 + \frac{A}{(x+1)} + \frac{B}{(x-4)}$, where A and B are constants to be found.

Divide: $2x^2 - x - 8 = 2(x^2 - 3x - 4) + (5x)$

So the fraction is $2 + \frac{5x}{(x+1)(x-4)}$

$A(x-4) + B(x+1) \equiv 5x$

$x = -1$ gives $A = 1$; $x = 4$ gives $B = 4$

Answer: A = 1, B = 4

8 Range of horizontal force on a rough slope (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 2 kg particle is in equilibrium on a rough plane inclined at α, $\tan\alpha = \frac{4}{3}$, $\mu = 0.2$. A horizontal force of P N, in the vertical plane through a line of greatest slope, pushes it towards the plane. Find the least and greatest values of P. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = 0.8$, $\cos\alpha = 0.6$, weight W = 19.6 N. Normal reaction $R = W\cos\alpha + P\sin\alpha$

Least P: about to slide down, friction $F = \mu R$ up the slope

$P\cos\alpha + \mu(W\cos\alpha + P\sin\alpha) = W\sin\alpha \rightarrow P = W(\sin\alpha - \mu\cos\alpha) \div (\cos\alpha + \mu\sin\alpha) = 17.5$

Greatest P: about to slide up, friction down the slope

$P\cos\alpha = W\sin\alpha + \mu(W\cos\alpha + P\sin\alpha) \rightarrow P = W(\sin\alpha + \mu\cos\alpha) \div (\cos\alpha - \mu\sin\alpha) = 41.0$

Answer: least 17.5 N, greatest 41.0 N

9 Angle between vectors (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C have coordinates (0, −1, 3), (2, 3, 4) and (3, −1, 0).

Find angle ABC, in degrees to 1 d.p.

$BA = \begin{pmatrix} -2 \\ -4 \\ -1 \end{pmatrix}$, $BC = \begin{pmatrix} 1 \\ -4 \\ -4 \end{pmatrix}$

$BA \cdot BC = 18$

$|BA| = \sqrt{21}$, $|BC| = \sqrt{33}$

$\cos ABC = \frac{18}{\sqrt{21} \times \sqrt{33}} = 0.6838$, so angle ABC = 46.9°

Answer: 46.9°

10 Binomial expansion: finding n and k (All boards)

In the expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is 33 and the coefficient of x^2 is 495.

Find n and k.

$nk = 33$ and $\frac{n(n-1)}{2}k^2 = 495$

$(n-1)k = \frac{2 \times 495}{33} = 30$ (dividing the second equation by the first)

$n\,k - (n-1)\,k = k = 33 - 30 = 3$

$n = \frac{33}{3} = 11$

Answer: n = 11, k = 3