Day 309 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Triangle on a coordinate grid (All boards)

The diagram shows triangle ABC.
(a) Find the equation of the line through C perpendicular to AB.
(b) Hence find the coordinates of D, the foot of the perpendicular from C to AB, and the area of triangle ABC.

Gradient AB = 0, so gradient of the perpendicular = undefined (vertical)
Through C: x = 1
Line AB: $y - 4 = 0(x - -3)$. Solve to get D = (1, 4)
Area = ½ × AB × CD = 20
Answer: (a) x = 1 (b) D(1, 4), area 20

2 Exact definite integrals of standard functions (All boards)

Find the exact value of

$\int_1^5 \frac{4}{2x+1}\,dx$.

Integrate: $[\frac{4}{2}\ln(2x+1)]$

Substitute the limits and subtract: 2 ln 11 − 2 ln 3
Answer: 2 ln 11 − 2 ln 3

3 Combined inequalities (All boards)

(a) Solve $4x - 3 < -5$.
(b) Solve $x^2 + 2x - 24 > 0$.
(c) Hence find the values of x satisfying both inequalities.

(a) $4x - 3 < -5$ so $x < -\frac{1}{2}$
(b) $(x + 6)(x - 4) > 0$: critical values −6 and 4
(b) gives $x < -6$ or $x > 4$
(c) Overlap of the two sets.
Answer: (a) $x < -\frac{1}{2}$ (b) $x < -6$ or $x > 4$ (c) $x < -6$

4 Interpolation for median and percentiles (All boards)

The table summarises the mass, m kg, of parcels of 49 items.
Use linear interpolation to estimate the upper quartile.

Position = 0.75 × 49 = 36.75
Cumulative frequency before the class 80 ≤ m < 100 is 25
Estimate = $80 + \frac{36.75 - 25}{14} \times 20$
= 96.8
Answer: 96.8

5 Differentiating sec, cosec, tan and cot (All boards)

y = x tan(3x).

Find the exact gradient at $x = \frac{\pi}{9}$.

Product rule: $\frac{dy}{dx} = \tan(3x) + x(3\sec^2(3x))$

At $x = \frac{\pi}{9}$: $\tan(3x) = \sqrt{3}$ and the derivative term = 12

Gradient = $\sqrt{3} + \frac{\pi}{9} \times 12 = \frac{3\sqrt{3} + 4\pi}{3}$

Answer: gradient $\frac{3\sqrt{3} + 4\pi}{3}$

Day 309 **A Level Maths: A Tier 10-a-day: Model Solutions**

6 Normal approximation to the binomial (All boards)

X ~ B(40, 0.3). Use a normal approximation to estimate P(X ≥ 16) to 3 significant figures.

np = 12, npq = 8.4, so Y ~ N(12, 8.4)
With continuity correction: P(Y > 15.5)
$z = \frac{15.5 - 12}{2.90} = 1.21$, P(Z > 1.21) = 0.114
Answer: 0.114

7 Separable differential equations (All boards)

Solve the differential equation
$x\frac{dy}{dx} = 2y,$
given that y = 3 when x = 1 (x > 0, y > 0). Give y in terms of x.

Separate: $\int \frac{1}{y}\,dy = \int \frac{2}{x}\,dx$, so ln y = 2 ln x + C
x = 1, y = 3 gives C = ln 3
ln y = ln($3x^2$), so $y = 3x^2$
Answer: $y = 3x^2$

8 Constant speed against friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A crate of mass 4 kg is pulled at constant speed along rough horizontal ground, μ = 0.2, by a force of P N at angle θ above the horizontal, $\tan\theta = \frac{3}{4}$. Find P and the normal reaction. Take g = 9.8. Give answers to 3 s.f. if not exact.

Constant speed, so the resultant force is zero. sin θ = 0.6, cos θ = 0.8
Vertical: R = 4 g − P sin θ
Horizontal: P cos θ = μR = 0.2(4 g − P sin θ)
P(0.8 + 0.12) = 0.2×4 g
P = 8.52, R = 34.1
Answer: P = 8.52 N, R = 34.1 N

9 Circle and line: chord length (AQA, Edexcel, OCR, MEI, Eduqas)

The line 3x + 4y = 9 meets the circle $(x + 3)^2 + (y - 3)^2 = 9$ at the points A and B. Find the exact length of AB.

Centre (−3, 3), radius 3. Distance from centre to line:
$d = \frac{|-6|}{5} = \frac{6}{5}$
$\text{Half-chord}^2 = 9 - (\frac{6}{5})^2 = \frac{189}{25}$
$AB = 2\sqrt{\frac{189}{25}}$
Answer: $\frac{30\sqrt{21}}{25}$

10 Projectile: perpendicular directions (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from the ground at 14 m s^{-1} at angle α above the horizontal, $\tan\alpha = \frac{15}{8}$. At time T s, while in flight, it is moving perpendicular to its initial direction. Find (a) T, (b) the height of the ball then. Take g = 9.8. Give answers to 3 s.f. if not exact.

Initial velocity $(\frac{112}{17}, \frac{210}{17})$. At time T the velocity is $(\frac{112}{17}, \frac{210}{17} - 9.8T)$.

Perpendicular: scalar product = 0: $(\frac{112}{17})^2 + \frac{210}{17}(\frac{210}{17} - 9.8T) = 0$

$14^2 - 9.8T \times \frac{210}{17} = 0$ (using $\sin^2\alpha + \cos^2\alpha = 1$)

(a) $T = 14 \div (9.8 \times \frac{15}{17}) = \frac{34}{21}$

(b) $h = \frac{210}{17} \times \frac{34}{21} - 4.9 \times \frac{34}{21}^2 = \frac{322}{45}$

Answer: (a) T = 1.62 s (b) 7.16 m