**Day 308** **A Level Maths: A Tier 10-a-day: Model Solutions**

## 1 Transformations of a turning point (All boards)

The curve $y = f(x)$ has a maximum point at $(-1, 2)$.
Find the coordinates of the corresponding point on:
(a) $y = f(x - 2)$
(b) $y = f(-x)$
(c) $y = -f(x)$

(a) $y = f(x - 2)$: $(1, 2)$
(b) $y = f(-x)$: $(1, 2)$
(c) $y = -f(x)$: $(-1, -2)$
A stretch or translation moves the point in the stated direction only.
**Answer: (a) (1, 2); (b) (1, 2); (c) $(-1, -2)$**

## 2 Stationary points with trigonometric and exponential terms (All boards)

$y = e^{2x} - 14e^{x} + 24x$.

Show that the stationary points occur where $e^{x} = 3$ and $e^{x} = 4$, and find the exact y-coordinate of each, with its nature.

$\frac{dy}{dx} = 2e^{2x} - 14e^{x} + 24 = 2(e^{x} - 3)(e^{x} - 4)$
So $e^{x} = 3$ or $e^{x} = 4$, i.e. $x = \ln 3$ or $x = \ln 4$
$\frac{d^2y}{dx^2} = 4e^{2x} - 14e^{x} = e^{x}(4e^{x} - 14)$ changes sign: negative at the smaller root, positive at the larger
**Answer: $x = \ln 3$, $y = 24 \ln 3 - 33$ (maximum); $x = 2 \ln 2$, $y = 48 \ln 2 - 40$ (minimum)**

## 3 Completing the square (All boards)

$f(x) = -x^2 + 2x + 5$.

(a) Write $f(x)$ in the form $a(x + p)^2 + q$.
(b) State the maximum value of $f(x)$ and the value of $x$ at which it occurs.
(c) Solve $f(x) = 0$, giving surds in simplest form.

$f(x) = -(x - 1)^2 + 6$
(b) maximum value 6 when $x = 1$
(c) $-(x - 1)^2 = -6$, so $(x - 1)^2 = 6$
$x - 1 = \pm\sqrt{6}$
**Answer: (a) $-(x - 1)^2 + 6$ (b) maximum 6 at $x = 1$ (c) $x = 1 \pm \sqrt{6}$**

## 4 Lift and cable tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 500 kg carries a person of mass 50 kg. The cable tension is 5610 N and the lift is accelerating. Find (a) the magnitude and direction of the acceleration, (b) the force of the floor on the person. Take $g = 9.8$.

Whole system (mass 550 kg), taking upwards as positive: $T - 550\,g = 550\,a$
$5610 - 5390 = 550\,a$, so $a = 0.4$
(a) a is positive, so the acceleration is 0.4 m $s^{-2}$ upwards
(b) Person: $R - 50\,g = 50\,a$, so $R = 50(9.8 + 0.4)$
**Answer: (a) 0.4 m $s^{-2}$ upwards (b) 510 N**

## 5 Recurrence relations: unknown first term (All boards)

A sequence is defined by $u_1 = k$ and $u_{n+1} = 3u_n + 2$.

Given that $u_3 = 71$, find (a) the value of k, (b) $\sum_{r=1}^{4} u_r$.

$u_2 = 3k + 2$, $u_3 = 3(3k + 2) + 2$
$9k + 8 = 71$ so $k = 7$
Terms: 7, 23, 71, 215
Sum = 316
**Answer: (a) $k = 7$ (b) 316**

## 6 Beam on two supports (AQA, Edexcel, OCR, MEI)

A uniform beam AB rests on supports at C and D, with a particle attached at A. Find the reactions at C and D. Take g = 9.8.

Moments about C: $R_D \times 7 = 20\,g \times 3.5 - 8\,g \times 0.5$

$R_D = 92.4$

Resolving vertically: $R_C + R_D = 28\,g = 274.4$

$R_C = 274.4 - 92.4$

**Answer: $R_C = 182$ N, $R_D = 92.4$ N**

## 7 Parametric differentiation with trigonometry (All boards)

A curve has parametric equations $x = 3\sin t$, $y = 2\cos 2t$.

Find $\frac{dy}{dx}$ in terms of t and the exact equation of the tangent at the point where $t = \frac{\pi}{4}$, in the form $y = mx + c$.

$$\frac{dy}{dx} = \frac{dy}{dt \div dx/dt} = -\frac{4\sin 2t}{3\cos t}$$

$t = \frac{\pi}{4}$: gradient $m = -\frac{4\sqrt{2}}{3}$, point $(\frac{3\sqrt{2}}{2}, 0)$

$c = y - mx = 4$

**Answer: $\frac{dy}{dx} = -\frac{4\sin 2t}{3\cos t}$; $y = -\frac{4\sqrt{2}}{3}x + 4$**

## 8 Binomial in two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Bulbs are defective with probability 0.25, independently. A pack of 5 bulbs is rejected if it has at least 2 defective. From 8 such packs, find the probability that exactly 4 are rejected (3 s.f.).

$Y \sim B(5, 0.25)$. $P(\text{rejected}) = P(Y \geq 2) = 1 - 0.633 = 0.367$

$W \sim B(8, 0.367)$ is the number rejected

$P(W = 4) = C(8, 4)(0.367)^4(1 - 0.367)^4 = 0.204$

**Answer: 0.204**

## 9 Small angle approximations: solving an equation (AQA, Edexcel, OCR, MEI, Eduqas)

$\theta$ is small and measured in radians. Use small angle approximations to show that

$5\sin 3\theta + \cos 2\theta = 2$

can be written as $a\theta^2 + b\theta + c = 0$ with integers a, b, c. Hence find the approximate solution with $0 < \theta < 0.8$, to 3 s.f.

$\sin 3\theta \approx 3\theta$, $\cos 2\theta \approx 1 - \frac{4}{2}\theta^2$

$5 \times 3\theta + 1(1 - \frac{4}{2}\theta^2) = 2$

Multiply by 2 and rearrange: $4\theta^2 - 30\theta + 2 = 0$

Divide through by 2: $2\theta^2 - 15\theta + 1 = 0$

$\theta = \frac{15 \pm \sqrt{217}}{4}$, so $\theta = 0.0673$

**Answer: $a = 2$, $b = -15$, $c = 1$; $\theta \approx 0.0673$**

## 10 Type I and Type II errors (AQA, Edexcel, OCR, MEI, Eduqas)

$X \sim B(21, p)$. $H_0$: $p = 0.3$ is tested against $H_1$: $p > 0.3$ using a 5% significance level.

(a) Find the critical region. (b) State the probability of a Type I error.

(c) Find the probability of a Type II error when $p = 0.45$ (3 s.f.).

(a) Largest/smallest region with probability $\leq 0.05$ under $H_0$: $X \geq 11$

(b) Type I error = $P(\text{reject } H_0 \mid p = 0.3) = 0.0264$

(c) Type II error = P(not in critical region | $p = 0.45$)

$P(X \leq 10)$ when $p = 0.45$: 0.679

**Answer: (a) $X \geq 11$ (b) 0.0264 (c) 0.679**