Day 307 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Venn diagrams and conditional probability (All boards)

P(A) = 0.55, P(B) = 0.35 and P(A ∩ B) = 0.05.
Find
(a) P(A ∪ B)
(b) P(B | A)
(c) P(A | B′)

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.55 + 0.35 - 0.05 = 0.85$

(b) $P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{0.05}{0.55} = \frac{1}{11}$

(c) $P(A \mid B') = \frac{P(A \cap B')}{P(B')} = \frac{0.55 - 0.05}{1 - 0.35} = \frac{10}{13}$

Answer: (a) 0.85 (b) $\frac{1}{11}$ (c) $\frac{10}{13}$

2 Exact definite integrals of standard functions (All boards)

Find the exact value of

$\int_2^3 \frac{2}{2x+1}\,dx$.

Integrate: $[\frac{2}{2}\ln(2x+1)]$

Substitute the limits and subtract: $\ln 7 - \ln 5$

Answer: $\ln 7 - \ln 5$

3 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas)

Express $\frac{-3+4\sqrt{2}}{1+\sqrt{2}}$ in the form $p + q\sqrt{2}$, where p and q are rational numbers.

Multiply top and bottom by $1 - \sqrt{2}$.
Denominator: $(1+\sqrt{2})(1-\sqrt{2}) = -1$
Numerator: $(-3+4\sqrt{2})(1-\sqrt{2}) = -11 + 7\sqrt{2}$
So the expression is $\frac{-11+7\sqrt{2}}{-1}$.

Answer: $11 - 7\sqrt{2}$

4 Equilibrium on a slope: range of force (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 2 kg particle rests on a rough plane inclined at α, $\tan\alpha = \frac{12}{5}$, μ = 0.5. A force of P N acts up a line of greatest slope and the particle stays at rest. Find the range of values of P. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = \frac{12}{13}$, $\cos\alpha = \frac{5}{13}$, $R = 2g\cos\alpha = \frac{98}{13}$ N

Limiting friction = $\mu R = \frac{49}{13}$ N

P smallest: friction acts up the slope: $P + \mu R = 2g\sin\alpha$, $P = \frac{1176}{65} - \frac{49}{13}$

P greatest: friction acts down the slope: $P = \frac{1176}{65} + \frac{49}{13}$

Answer: $14.3\ \text{N} \leq P \leq 21.9\ \text{N}$

5 Finding mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(\mu, \sigma^2)$, with P(X < 42) = 0.1587 and P(X > 45) = 0.3085.
Find μ and σ.

z-values: 42: z = −1.00; 45: z = 0.500
$42 = \mu - 1.00\,\sigma$ and $45 = \mu + 0.500\,\sigma$
Subtract: $3 = 1.50\,\sigma$, so $\sigma = 2.00$
$\mu = 42 + 1.00 \times 2.00 = 44.0$

Answer: $\mu = 44$, $\sigma = 2$

6 Parametric equations: points on axes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 2t^2 - 3$, $y = t(t^2 - 16)$.
Find the coordinates of the points where the curve meets the x-axis.

$y = 0$: $t = 0$ or $t^2 = 16$, so $t = 0, 4, -4$
$t = 0$: $x = -3$
$t = \pm 4$: $x = 29$
Answer: $(-3, 0)$ and $(29, 0)$

7 Position vector and velocity (AQA, Edexcel, OCR, MEI, Eduqas)

A particle P has position vector $\mathbf{r} = (t^3 - t)\mathbf{i} + (2t^2 - 4t)\mathbf{j}$ metres at time t s. Find (a) the speed of P when $t = 4$, (b) the value of t when P is moving parallel to $\mathbf{i}$. Give answers to 3 s.f. if not exact.

$\mathbf{v} = d\mathbf{r}/dt = (3t^2 - 1)\mathbf{i} + (4t - 4)\mathbf{j}$
(a) $t = 4$: $\mathbf{v} = 47\mathbf{i} + 12\mathbf{j}$
Speed $= \sqrt{(47^2 + 12^2)} = \sqrt{2353}$
(b) Parallel to $\mathbf{i}$ means the $\mathbf{j}$ component is zero: $4t - 4 = 0$, so $t = 1$
Answer: (a) 48.5 m s^{-1} (b) $t = 1$

8 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of lg y against lg x for $y = A x^n$.
Find the value of n and the value of A.

$\lg y = \lg A + n \lg x$: gradient n, intercept $\lg A$
$n = \dfrac{10 - 6}{3 - 1} = 2$
At (1, 6): $\lg A = 6 - 2 \times 1 = 4$, so $A = 10^4 = 10000$
Answer: $n = 2$, $A = 10000$

9 Partial fractions: repeated factor (All boards)

Express $\dfrac{2x^2 - 17x - 15}{(x - 3)(x + 1)^2}$ in the form $\dfrac{A}{(x - 3)} + \dfrac{B}{(x + 1)} + \dfrac{C}{(x + 1)^2}$, and find A, B and C.

$2x^2 - 17x - 15 \equiv A(x + 1)^2 + B(x - 3)(x + 1) + C(x - 3)$
$x = 3$: $-48 = A(4)^2$ so $A = -3$
$x = -1$: $4 = C(-4)$ so $C = -1$
Coefficient of x^2: $2 = A + B$ so $B = 5$
Answer: $A = -3$, $B = 5$, $C = -1$

10 Exponential growth and decay modelling (All boards)

The value of an investment is modelled by $V = 100\,e^{kt}$, where t is time in years.
After 3 years, $V = 200$.
Find (a) the exact value of k, (b) the time for V to reach 5000, to 3 s.f.

$100\,e^{3k} = 200$, so $e^{3k} = 2$ and $k = \dfrac{\ln 2}{3}$
$k = 0.231$ (3 s.f.)
$100\,e^{kt} = 5000$ gives $kt = \ln 50$, so $t = \dfrac{\ln 50}{k} = 16.9$ years
Answer: (a) $k = \dfrac{\ln 2}{3}$ (b) $t = 16.9$ years