Day 306 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Index equations solved with logarithms (All boards)

Solve $3^{2x+2} = 7^{3x+3}$.
Give your answer to 3 significant figures.

Take logs: $(2x + 2) \ln 3 = (3x + 3) \ln 7$
Collect x terms: $x(2 \ln 3 - 3 \ln 7) = 3 \ln 7 - 2 \ln 3$
$x = \frac{3 \ln 7 - 2 \ln 3}{2 \ln 3 - 3 \ln 7}$
$x = -1.00$
Answer: $x = -1.00$

2 Critical values for a binomial distribution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(15, 0.3)$.
Find the smallest value of c such that $P(X \geq c) < 0.1$.

$P(X \geq 8) = 1 - P(X \leq 7) = 0.0500$
$P(X \geq 7) = 1 - P(X \leq 6) = 0.131$
$0.0500 < 0.1$ but $0.131 > 0.1$
Answer: $c = 8$

3 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that $(n + 4)^2 - (n - 4)^2$ is a multiple of 16 for every integer n.

Difference of two squares:
$(n + 4)^2 - (n - 4)^2 = (n + 4 - n + 4)(n + 4 + n - 4)$
$= 8 \times 2n = 16n$
16n is a multiple of 16.
Answer: 16n, a multiple of 16

4 Finding an unknown standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X is normally distributed with mean 48 and standard deviation σ.
Given that $P(X > 66) = 0.0668$, find σ to 3 significant figures.

$P(Z > z) = 0.0668$, so $z = 1.50$
$\frac{66 - 48}{\sigma} = 1.50$
$\sigma = \frac{18}{1.50} = 12.0$
Answer: $\sigma = 12.0$

5 Projectile from and to the ground (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from a point O on horizontal ground at 13 m s^{-1} at angle α above the horizontal, $\tan \alpha = \frac{5}{12}$, and lands at A. Find (a) the time of flight, (b) the greatest height, (c) OA. Take g = 9.8. Give answers to 3 s.f. if not exact.

Horizontal component $= 13 \times \frac{12}{13} = 12$; vertical component $= 13 \times \frac{5}{13} = 5$

(a) Vertical: $0 = 5t - 4.9t^2$, so $t = \frac{50}{49}$

(b) At the top: $0 = 5^2 - 2 \times 9.8 \times H$, so $H = \frac{125}{98}$

(c) $OA = 12 \times \frac{50}{49}$

Answer: (a) 1.02 s (b) 1.28 m (c) 12.2 m

6 Collinear points and ratios (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A, B and C have position vectors $\begin{pmatrix}-4\\-3\\5\end{pmatrix}$, $\begin{pmatrix}-2\\0\\7\end{pmatrix}$ and $\begin{pmatrix}k\\6\\11\end{pmatrix}$ relative to O, where k is a constant. Given that A, B and C are collinear, find k and the ratio AB : BC.

$\overrightarrow{AB} = \begin{pmatrix}2\\3\\2\end{pmatrix}$, and $\overrightarrow{AC} = \begin{pmatrix}k+4\\9\\6\end{pmatrix}$

$\overrightarrow{AC} = \lambda\overrightarrow{AB}$: component 1 gives $6 = 2\lambda$, so $\lambda = 3$

Hence $k = -4 + 3 \times 2 = 2$

$\overrightarrow{AC} = 3\overrightarrow{AB}$, so $\overrightarrow{BC} = 2\overrightarrow{AB}$: AB : BC = 1 : 2

Answer: k = 2, AB : BC = 1 : 2

7 Exponential equations: quadratic in b^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $3^{2x} - 10 \times 3^x + 9 = 0$.

Let $y = 3^x$. Then $y^2 - 10y + 9 = 0$

$(y - 1)(y - 9) = 0$

$y = 1$ or $y = 9$

$3^x = 1$ gives $x = 0$; $3^x = 9$ gives $x = 2$

Answer: x = 0 or x = 2

8 Connected particles (rough table, string slack) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (4 kg) lies on a rough table, $\mu = 0.4$, joined by a light string over a smooth pulley to particle B (10 kg), 1.2 m above the floor. Released from rest; B lands without rebounding. Find (a) the acceleration of A before B lands, (b) the further distance A moves. Take $g = 9.8$.

B: $10g - T = 10a$; A: $T - 0.4 \times 4g = 4a$

Adding: $(10 - 0.4 \times 4)g = 14a$, so $a = 5.88$

Speed of B and A as B lands: $v^2 = 2 \times 5.88 \times 1.2 = 14.112$

After B lands, A decelerates at $\mu g = 0.4 \times 9.8 = 3.92$

$0 = v^2 - 2(\mu g)s$, $s = 14.112 \div 7.84$

Answer: (a) 5.88 m s^{-2} (b) 1.8 m

9 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $x^2 - 2xy + 3y^2 = 8$.

Find the coordinates of the points where the tangent is parallel to the x-axis.

$\frac{dy}{dx} = -\frac{2x - 2y}{-2x + 6y}$

Horizontal tangent: $2x - 2y = 0$, so $y = x$

Substitute: $2x^2 = 8$, so $x^2 = 4$ and $x = \pm 2$

Points: (2, 2) and (−2, −2)

Answer: (2, 2) and (−2, −2)

10 Geometric sequences: finding a and r (All boards)

The 4th term of a geometric sequence is −48 and the 7th term is 384.

Find the common ratio and the first term.

Find the sum of the first 5 terms.

$r^3 = \frac{384}{-48} = -8$ so $r = -2$

$ar^3 = -48$ so $a = 6$

$S_5 = \frac{a(r^5 - 1)}{(r - 1)} = \frac{6((-32) - 1)}{-3}$

Answer: r = −2, a = 6, S_5 = 66