Day 305 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Double and triple angle values (All boards)

θ is acute and $\tan\theta = \frac{5}{2}$.

Show that $\cos 2\theta = -\frac{21}{29}$ and find the exact value of $\sin 2\theta$.

$$\cos 2\theta = \frac{1 - \tan^2\theta}{1 + \tan^2\theta} = \frac{1 - \frac{5}{2}^2}{1 + \frac{5}{2}^2} = -\frac{21}{29}$$

$$\sin 2\theta = \frac{2\tan\theta}{1 + \tan^2\theta} = \frac{2 \times \frac{5}{2}}{1 + \frac{5}{2}^2} = \frac{20}{29}$$

Answer: $\cos 2\theta = -\frac{21}{29}$, $\sin 2\theta = \frac{20}{29}$

2 Forces in equilibrium with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

Forces $F_1 = (-6\mathbf{i} - 9\mathbf{j})$ N, $F_2 = (-6\mathbf{i} + 8\mathbf{j})$ N and $F_3 = (a\mathbf{i} + b\mathbf{j})$ N keep a 3 kg particle in equilibrium. Find (a) a and b. F_2 is then removed; (b) find the magnitude of the acceleration. Give answers to 3 s.f. if not exact.

(a) Equilibrium: $F_1 + F_2 + F_3 = 0$

$\mathbf{i}$: $-6 + (-6) + a = 0$, so $a = 12$

$\mathbf{j}$: $-9 + 8 + b = 0$, so $b = 1$

(b) The resultant is now $F_1 + F_3 = -F_2$, of magnitude $\sqrt{(-6^2 + 8^2)} = 10$ N

$a = 10 \div 3$

Answer: (a) $a = 12$, $b = 1$ (b) 3.33 m s^{-2}

3 Completing the square (All boards)

$f(x) = x^2 - 6x + 6$.

(a) Write f(x) in the form $a(x + p)^2 + q$.

(b) State the minimum value of f(x) and the value of x at which it occurs.

(c) Solve $f(x) = 0$, giving surds in simplest form.

$f(x) = (x - 3)^2 - 3$

(b) minimum value -3 when $x = 3$

(c) $(x - 3)^2 = 3$, so $(x - 3)^2 = 3$

$x - 3 = \pm\sqrt{3}$

Answer: (a) $(x - 3)^2 - 3$ (b) minimum -3 at $x = 3$ (c) $x = 3 \pm \sqrt{3}$

4 Product moment correlation coefficient (AQA, Edexcel, OCR, MEI, Eduqas)

For n = 10 pairs of (x, y): $\Sigma x = 180$, $\Sigma y = 572$,

$\Sigma x^2 = 3736$, $\Sigma y^2 = 36062$, $\Sigma xy = 11522$.

Calculate the product moment correlation coefficient, and describe the correlation.

$$S_{xx} = 3736 - \frac{180^2}{10} = 496$$

$$S_{yy} = 36062 - \frac{572^2}{10} = 3340$$

$$S_{xy} = 11522 - \frac{180 \times 572}{10} = 1230$$

$$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = 0.952$$

This is strong positive correlation

Answer: r = 0.952, positive correlation

5 Binomial expansion: rational powers (All boards)

Expand $(1 + x)^{-\frac{3}{2}}$ in ascending powers of x up to and including the term in x^2.
State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \frac{n(n-1)(n-2)}{3!}y^3 + \ldots$ with $n = -\frac{3}{2}$, $y = x$

Coefficients of 1, x, x^2: 1, $-\frac{3}{2}$, $\frac{15}{8}$

Valid when $|x| < 1$, i.e. $|x| < 1$

Answer: $1 - \frac{3}{2}x + \frac{15}{8}x^2$; $|x| < 1$

6 Estimating mean and standard deviation (All boards)

The table summarises the mass, m grams, of eggs.
Estimate the mean and standard deviation (3 s.f.).

Midpoints: 2.5, 7.5, 12.5, 17.5, 22.5 (n = 52)

$\Sigma fx = 515$, so mean $= \frac{515}{52} = 9.90$

$\Sigma fx^2 = 7675$

sd $= \sqrt{\frac{7675}{52} - (9.90)^2} = 7.04$

Answer: mean 9.90, sd 7.04

7 R-form and solving a sin x + b cos x = c (All boards)

(a) Express $9 \sin x - 12 \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give α to 1 d.p.
(b) Solve $9 \sin x - 12 \cos x = -9$ for $0° \leq x < 360°$, giving answers to 1 d.p.

$R^2 = 9^2 + 12^2 = 225$, so $R = 15$; $\tan \alpha = \frac{12}{9}$ gives $\alpha = 53.1°$

$15 \sin(x - 53.1°) = -9$, so $\sin(x - 53.1°) = -\frac{3}{5}$

Principal value $-36.9°$, then use the symmetry of the sine graph
$x = 16.3°, 270.0°$

Answer: (a) R = 15, α = 53.1° (b) x = 16.3°, 270.0°

8 Recurrence relations: limits (All boards)

$u_1 = 36$ and $u_{n+1} = \frac{1}{2}u_n + 9$.

(a) Find the limit L of the sequence as $n \to \infty$.
(b) Find the least n for which $|u_n - L| < 0.01$.

(a) $L = \frac{1}{2}L + 9$ gives $L = \frac{9}{1 - \frac{1}{2}} = 18$

(b) $u_n - L = (u_1 - L)\, r^{n-1} = 18 \times \left(\frac{1}{2}\right)^{n-1}$

$|18| \times \left(\frac{1}{2}\right)^{n-1} < \frac{1}{100}$

Check: n = 11 gives 0.01758, n = 12 gives 0.00879

Answer: (a) L = 18 (b) n = 12

9 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of ln y against ln x for $y = A x^n$.
Find the value of n and the exact value of A.

ln y = ln A + n ln x: gradient n, intercept ln A

$n = \frac{4 - 3}{4 - 2} = \frac{1}{2}$

At (2, 3): $\ln A = 3 - \frac{1}{2} \times 2 = 2$, so $A = e^2$

Answer: $n = \frac{1}{2}$, $A = e^2$

10 Connected particles (smooth table) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (6 kg) rests on a smooth table, joined by a light string over a smooth pulley at the edge to particle B (1 kg), hanging 1.2 m above the floor. Released from rest. Find (a) the tension, (b) the speed of B as it hits the floor. Take g = 9.8. Give answers to 3 s.f. if not exact.

B: $1g - T = 1a$

A: $T = 6a$

Adding: $1g = 7a$, so $a = 1.4$

(a) $T = 6 \times 1.4 = 8.4$

(b) $v^2 = 0 + 2 \times 1.4 \times 1.2 = 3.36$

Answer: (a) 8.4 N (b) 1.83 m s^{-1}