Day 304 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Interpolation for median and percentiles (All boards)

The table summarises the journey time, t minutes of 55 items.
Use linear interpolation to estimate the median.

Position = 0.5 × 55 = 27.5
Cumulative frequency before the class 60 ≤ t < 65 is 20

Estimate = $60 + \frac{27.5 - 20}{12} \times 5$

= 63.1

Answer: 63.1

2 Double angle equations (All boards)

Solve $\cos 2\theta + 2\cos\theta = -1$
for $-180° \le \theta \le 180°$.

$\cos 2\theta \equiv 2\cos^2\theta - 1$: $2\cos^2\theta + 2\cos\theta = 0$
Solve the quadratic: $\cos\theta = 0$ or $\cos\theta = -1$
$\cos\theta = 0$: $\theta = -90°, 90°$
$\cos\theta = -1$: $\theta = -180°, 180°$
Answer: $\theta = -180°, -90°, 90°, 180°$

3 Constant speed against friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A crate of mass 10 kg is pulled at constant speed along rough horizontal ground, $\mu = 0.5$, by a force of P N at angle θ above the horizontal, $\tan\theta = \frac{12}{5}$. Find P and the normal reaction. Take g = 9.8. Give answers to 3 s.f. if not exact.

Constant speed, so the resultant force is zero. $\sin\theta = \frac{12}{13}$, $\cos\theta = \frac{5}{13}$

Vertical: $R = 10g - P\sin\theta$
Horizontal: $P\cos\theta = \mu R = 0.5(10g - P\sin\theta)$
$P(\frac{5}{13} + \frac{6}{13}) = 0.5 \times 10g$
P = 57.9, R = 44.5
Answer: P = 57.9 N, R = 44.5 N

4 Binomial mean and variance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X has a binomial distribution with mean 15 and variance 7.5.
(a) Find n and p.
(b) Find P(X = 13) to 3 significant figures.

(a) np = 15 and npq = 7.5
$q = \frac{npq}{np} = \frac{7.5}{15} = 0.5$, so p = 0.5
$n = \frac{15}{0.5} = 30$
(b) $P(X = 13) = C(30, 13)(0.5)^{13}(0.5)^{17} = 0.112$
Answer: (a) n = 30, p = 0.5 (b) 0.112

5 Equation after a sequence of transformations (All boards)

The curve with equation $y = x^2 + x + 1$ is transformed by a reflection in the x-axis, followed by a reflection in the y-axis. Find the equation of the image curve.

Reflect in x-axis: y becomes −y: $y = -x^2 - x - 1$
Reflect in y-axis: replace x by −x: $y = -x^2 + x - 1$
Answer: $y = -x^2 + x - 1$

6 Equations with sec x and tan x (All boards)

Solve $3\sec^2\theta - 6\tan\theta - 12 = 0$
for $0° \le \theta \le 360°$. Give answers to 1 d.p.

Use $\sec^2\theta \equiv 1 + \tan^2\theta$: $3\tan^2\theta - 6\tan\theta - 9 = 0$
$\tan\theta = 3$ or $\tan\theta = -1$
$\tan\theta = 3$: $\theta = 71.6°, 251.6°$
$\tan\theta = -1$: $\theta = 135.0°, 315.0°$
Answer: $\theta = 71.6°, 135.0°, 251.6°, 315.0°$

7 Binomial expansion: terms with negative powers of x (All boards)

Find the coefficient of x^{-2} in the expansion of
$(2x^3 - \frac{1}{x^2})^6$.

General term: ${}^6C_r\,(2x^3)^{6-r}\,(-x^{-2})^r$
Power of x: $3(6 - r) - 2r = -2$
$18 - 5r = -2$ so $r = 4$
Coefficient = ${}^6C_4 \times 2^2 \times (-1)^4 = 60$
Answer: 60

8 Ladder in limiting equilibrium (AQA, Edexcel, OCR, MEI)

A uniform ladder AB (5 m, 25 kg) rests with A on rough ground, $\mu = 0.5$, and B against a smooth wall, B being 4 m above the ground. A man of mass 50 kg climbs the ladder. How far along the ladder from A can he climb before it slips?

Foot of ladder is $\sqrt{(5^2 - 4^2)} = 3$ m from the wall
Vertical: $R = (25 + 50)g$. Limiting friction: $F = \mu R = 0.5(25 + 50)g$
Horizontal: $S = F$ (wall reaction S)
Moments about A (man at distance x along ladder, horizontal distance $x \times 3/5$):
$S \times 4 = 25\,g \times 1.5 + 50\,g \times (3x \div 5)$
$37.5 \times 4 = 37.5 + 30x$ (g cancels), so $x = 3.75$
Answer: 3.75 m

9 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{-24x - 2}{(1 - 3x)(1 + 3x)}$.
(a) Express f(x) in partial fractions.
(b) Hence expand f(x) up to x^2.
(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 - 3x)} + \frac{B}{(1 + 3x)}$: $A = -5$, $B = 3$
(b) $-5(1 - 3x)^{-1} = -5(1 + 3x + 9x^2 \ldots)$
Add the series for both fractions.
(c) Need $|3x| < 1$ and $|3x| < 1$
Answer: (a) $\frac{-5}{(1 - 3x)} + \frac{3}{(1 + 3x)}$ (b) $-2 - 24x - 18x^2$ (c) $|x| < \frac{1}{3}$

10 Intersecting or skew lines (CIE (9709); Further Maths on other boards)

l_1: $\mathbf{r} = \begin{pmatrix}2\\4\\3\end{pmatrix} + \lambda\begin{pmatrix}-2\\0\\1\end{pmatrix}$

l_2: $\mathbf{r} = \begin{pmatrix}1\\4\\3\end{pmatrix} + \mu\begin{pmatrix}-3\\1\\-1\end{pmatrix}$

Determine whether l_1 and l_2 intersect. If they do, find the point of intersection.

Equate components: $2 - 2\lambda = 1 - 3\mu$; $4 + 0\lambda = 4 + 1\mu$; $3 + 1\lambda = 3 - 1\mu$
Solving two of the equations gives a unique (λ, μ)
which does not satisfy the third, so the lines do not meet
Answer: The lines do not intersect (skew)