Day 303 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Dividing a line in a ratio (2D position vectors) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A and B have position vectors $\begin{pmatrix}5\\3\end{pmatrix}$ and $\begin{pmatrix}-5\\3\end{pmatrix}$. The point P lies on AB with AP : PB = 3 : 2.

Find the position vector of P and the exact distance OP.

$\text{OP} = \text{OA} + \frac{3}{5}\text{AB} = \begin{pmatrix}5\\3\end{pmatrix} + \frac{3}{5}\begin{pmatrix}-10\\0\end{pmatrix}$

$= \begin{pmatrix}-1\\3\end{pmatrix}$

$\text{OP} = \sqrt{(-1)^2 + 3^2} = \sqrt{10} = \sqrt{10}$

Answer: $\begin{pmatrix}\mathbf{-1}\\\mathbf{3}\end{pmatrix}$**, OP =** $\sqrt{\mathbf{10}}$

2 Vertical motion under gravity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 34.3 m s^{-1} from a point h m above horizontal ground, and hits the ground 9 s later. Find (a) h, (b) the greatest height above the ground. Take g = 9.8.

(a) Take upwards as positive: s = −h, u = 34.3, a = −9.8, t = 9

$-h = 34.3\times9 - 4.9\times9^2$, so h = 88.2

(b) At the highest point v = 0: $0 = 34.3^2 - 2\times9.8\times s$

s = 60.025 m above the point of projection

Greatest height above the ground = 88.2 + 60.025

Answer: (a) 88.2 m (b) 148.225 m

3 Index equations solved with logarithms (All boards)

Solve $3^{2x+2} = 2^x$.

Give your answer to 3 significant figures.

Take logs: $(2x + 2)\ln 3 = x \ln 2$

Collect x terms: $x(2\ln 3 - \ln 2) = -2\ln 3$

$x = \frac{-2\ln 3}{2\ln 3 - \ln 2}$

x = −1.46

Answer: x = $\mathbf{-1.46}$

4 Stopping distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A driver at constant speed sees a hazard 77.5 m ahead. After a reaction time of 0.6 s she brakes with deceleration 5 m s^{-2} and stops exactly at the hazard. Find (a) her initial speed, (b) the total time from seeing the hazard to stopping.

Let the initial speed be v. Reaction distance = 0.6 v, braking distance = $v^2 \div (2\times5)$

$0.6\,v + v^2 \div 10 = 77.5$, so $v^2 + 6\,v - 775 = 0$

$(v - 25)(v + 31) = 0$, so v = 25

(b) Braking time = 25 ÷ 5 = 5 s; total = 0.6 + 5

Answer: (a) 25 m $\mathbf{s^{-1}}$ **(b) 5.6 s**

5 Foot of a perpendicular (All boards)

The line l has equation 3x − 3y = 6.

(a) Find the coordinates of the foot of the perpendicular from P(−3, −6) to l.

(b) Hence find the shortest distance from P to l, in exact form.

Gradient of l = 1, so perpendicular gradient = −1

Perpendicular through P: −3x − 3y = 27

Solve with l: $x = -\frac{7}{2}$, $y = -\frac{11}{2}$

$\text{Distance}^2 = (-3 - (-\frac{7}{2}))^2 + (-6 - (-\frac{11}{2}))^2 = \frac{1}{2}$

Answer: (a) $\mathbf{(-\frac{7}{2}, -\frac{11}{2})}$ **(b)** $\frac{\sqrt{2}}{2}$

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6 Hypothesis test for a normal mean (All boards)

The mean mass of a sack (kg) is normally distributed with standard deviation 3; it was previously 73.
A random sample of 9 has mean 71.1.
Test, at the 10% level, whether the mean has changed.

H_0: $\mu = 73$, H_1: $\mu \neq 73$

$\bar{X} \sim N(73, \frac{3^2}{9})$, $z = \frac{71.1 - 73}{\frac{3}{3}} = -1.90$

p-value = 0.0574 (two-tailed) < 0.1

Reject H_0: evidence that the mean has changed.

Answer: $z = -1.90$, $p = 0.0574$; reject H_0

7 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of lg y against lg x for $y = A x^n$.
Find the value of n and the value of A.

lg y = lg A + n lg x: gradient n, intercept lg A

$n = \frac{-3 - -1}{2 - 1} = -2$

At (1, −1): lg A = −1 − (−2) × 1 = 1, so $A = 10^1 = 10$

Answer: $n = -2$, $A = 10$

8 Circle: tangent at a point (AQA, Edexcel, OCR, MEI, Eduqas)

The circle $x^2 + y^2 + 2x - 2y - 23 = 0$ passes through P(−4, −3).
(a) Find the equation of the tangent at P, in the form ax + by = c.
(b) Find the coordinates of the point where this tangent crosses the x-axis.

Centre (−1, 1). Gradient of radius CP = $\frac{4}{3}$

Gradient of tangent = $-\frac{3}{4}$

$y + 3 = -\frac{3}{4}(x + 4)$: $3x + 4y = -24$

y = 0: x = −8

Answer: (a) $3x + 4y = -24$ (b) (−8, 0)

9 Recurrence relations: limits (All boards)

$u_1 = 0$ and $u_{n+1} = \frac{1}{3} u_n + 4$.

(a) Find the limit L of the sequence as $n \to \infty$.
(b) Find the least n for which $|u_n - L| < 0.001$.

(a) $L = \frac{1}{3} L + 4$ gives $L = \frac{4}{1 - \frac{1}{3}} = 6$

(b) $u_n - L = (u_1 - L) r^{n-1} = (-6) \times \frac{1}{3}^{n-1}$

$|-6| \times \frac{1}{3}^{n-1} < \frac{1}{1000}$

Check: n = 8 gives 0.00274, n = 9 gives 0.00091

Answer: (a) L = 6 (b) n = 9

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10 Coding mean and standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 40 values of x, the coded variable $y = \frac{x - 50}{2}$ gives

$\Sigma y = 60$ and $\Sigma y^2 = 450$.

Find the mean and standard deviation of x.

$\bar{y} = \frac{60}{40} = 1.5$

$\sigma_y^2 = \frac{450}{40} - 1.5^2 = 9$, so $\sigma_y = 3$

$x = 2y + 50$, so mean $= 2 \times 1.5 + 50 = 53$

Standard deviation $= 2 \times 3 = 6$

Answer: mean 53, sd 6