Day 302 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Vertical motion under gravity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 14.7 m s^{-1} from a point h m above horizontal ground, and hits the ground 5 s later. Find (a) h, (b) the greatest height above the ground. Take g = 9.8.

(a) Take upwards as positive: s = −h, u = 14.7, a = −9.8, t = 5
$-h = 14.7\times5 - 4.9\times5^2$, so h = 49
(b) At the highest point v = 0: $0 = 14.7^2 - 2\times9.8\times s$
s = 11.025 m above the point of projection
Greatest height above the ground = 49 + 11.025
Answer: (a) 49 m (b) 60.025 m

2 Geometric sequences: finding a and r (All boards)

The 3rd term of a geometric sequence is 24 and the 5th term is 96.
Find the common ratio and the first term.
Find the sum of the first 8 terms.

$r^2 = \frac{96}{24} = 4$ so r = −2

$a\,r^2 = 24$ so a = 6

$S_8 = \frac{a(r^8 - 1)}{(r - 1)} = \frac{6(256 - 1)}{-3}$

Answer: r = −2, a = 6, S_8 = −510

3 Trapezium rule with over or under estimate (AQA, Edexcel, OCR, MEI, Eduqas)

Use the trapezium rule with 4 strips to estimate $\int_1^5 \ln x\,dx$, giving your answer to 3 s.f.

State, with a reason, whether your estimate is an overestimate or an underestimate.

h = 1; y-values at x = 1, 2, 3, 4, 5: 0.0000, 0.6931, 1.0986, 1.3863, 1.6094

$T = \frac{1}{2}[0.0000 + 1.6094 + 2(0.6931 + 1.0986 + 1.3863)]$

T = 3.98
The curve is concave (y" < 0) on this interval, so the chords lie below the curve: underestimate
Answer: 3.98, an underestimate

4 Line and curve intersection (diagram) (All boards)

The diagram shows the curve $y = x^2 + 6x + 6$ and the line y = 2x + 3, meeting at A and B.
(a) Find the coordinates of A and B.
(b) Find the coordinates of the midpoint of AB.

$x^2 + 6x + 6 = 2x + 3$: $x^2 + 4x + 3 = 0$
(x + 3)(x + 1) = 0, x = −3 or −1
A(−3, −3), B(−1, 1)
Midpoint: average the coordinates
Answer: (a) A(−3, −3), B(−1, 1) (b) (−2, −1)

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5 Conditional probability without replacement (All boards)

A bag has 3 green and 5 yellow counters. Two are taken at random without replacement.
Given that the second counter is green, find the probability that the first was green.

P(green then green) = $\frac{3}{8} \times \frac{2}{7} = \frac{3}{28}$

P(yellow then green) = $\frac{5}{8} \times \frac{3}{7} = \frac{15}{56}$

P(second green) = $\frac{3}{28} + \frac{15}{56} = \frac{3}{8}$

P(first green | second green) = $\frac{\frac{3}{28}}{\frac{3}{8}}$

$= \frac{2}{7}$

Answer: $\frac{2}{7}$

6 Final seconds of a fall (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is dropped from rest from the top of a cliff. In the last 2 seconds before it hits the sea it falls 147 m. Find the height of the cliff. Take g = 9.8.

Let the total time of fall be t s. Then $s = 4.9t^2$ for the whole fall.
In the first $(t - 2)$ s the stone falls $4.9(t - 2)^2$.
$4.9t^2 - 4.9(t - 2)^2 = 147$
$4.9(4t - 4) = 147$, so $t = 8$
Height $= 4.9 \times 8^2 = 354.025$
Answer: 354.025 m

7 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^3 + 3x - 7$.
Use the Newton-Raphson method with $x_0 = 1.5$ to find x_1 and x_2, giving your answers to 4 d.p.

$f'(x) = 3x^2 + 3$

$x_1 = 1.5 - \frac{f(1.5)}{f'(1.5)} = 1.5 - \frac{0.8750}{9.7500} = 1.4103$

$x_2 = 1.4103 - \frac{f(1.4103)}{f'(1.4103)} = 1.4063$

Answer: $x_1 = 1.4103$, $x_2 = 1.4063$

8 Cubic inequalities (All boards)

Given that $(x + 4)$ is a factor of $f(x) = x^3 + 2x^2 - 23x - 60$, solve $f(x) > 0$.

Factorise: $f(x) = (x + 4)(x + 3)(x - 5)$
Roots at $x = -4, -3, 5$
Sketch the cubic (positive x^3 coefficient: negative, positive, negative, positive)
Read off where $f(x) > 0$.
Answer: $-4 < x < -3$ or $x > 5$

9 Correcting a recorded value (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

25 values have $\Sigma x = 1250$ and $\Sigma x^2 = 62900$.
One value was recorded as 58 but should have been 48.
Find the correct mean and standard deviation (3 s.f.).

$\Sigma x = 1250 - 58 + 48 = 1240$
$\Sigma x^2 = 62900 - 58^2 + 48^2 = 61840$
Mean $= \frac{1240}{25} = 49.6$
sd $= \sqrt{\frac{61840}{25} - (49.6)^2} = 3.67$
Answer: mean 49.6, sd 3.67

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10 Inverse trigonometric functions (AQA, Edexcel, OCR, MEI, Eduqas)

Find the exact value of $\sin(2 \arccos \frac{24}{25})$.

Let $\theta = \arccos \frac{24}{25}$, so $\cos \theta = \frac{24}{25}$ with θ acute

$\sin \theta = \sqrt{1 - \frac{24}{25}^2} = \frac{7}{25}$

$\sin 2\theta = 2\sin \theta \cos \theta = 2 \times \frac{7}{25} \times \frac{24}{25} = \frac{336}{625}$

Answer: $\mathbf{\frac{336}{625}}$