Day 301 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Polynomial division (All boards)

Find the quotient and the remainder when $2x^4 + x^3 + 6x^2 + 5$ is divided by $(x^2 + 1)$.

Quotient (matching leading terms in turn): $2x^2 + x + 4$

$(x^2 + 1)(2x^2 + x + 4) = 2x^4 + x^3 + 6x^2 + x + 4$

Remainder $= 2x^4 + x^3 + 6x^2 + 5 - (2x^4 + x^3 + 6x^2 + x + 4) = -x + 1$

Answer: quotient $2x^2 + x + 4$, remainder $-x + 1$

2 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

For a sample of 9 pairs of values of age of a car and its price, $r = -0.732$.
The critical value for a one-tailed test at the 5% level is 0.5822.
Test, at this level, whether there is negative correlation between the two variables.

$H_0: \rho = 0$, $H_1: \rho < 0$

Critical value -0.5822

$r = -0.732 < -0.5822$

This is in the critical region, so reject H_0.

There is evidence of negative correlation.

Answer: Reject H_0: evidence of correlation

3 Discriminant: range of k (All boards)

The equation $2x^2 + (2k - 4)x + (2k + 2) = 0$ has no real roots.
Find the set of possible values of k.

Discriminant: $b^2 - 4ac = (2k - 4)^2 - 4 \times 2(2k + 2)$

$= 4k^2 - 32k$

Roots of the quadratic in k: $k = 0$ and $k = 8$

Required: $b^2 - 4ac < 0$

Answer: $0 < k < 8$

4 Vertical motion under gravity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 19.6 m s^{-1} from a point h m above horizontal ground, and hits the ground 5 s later.
Find (a) h, (b) the greatest height above the ground. Take $g = 9.8$.

(a) Take upwards as positive: $s = -h$, $u = 19.6$, $a = -9.8$, $t = 5$

$-h = 19.6 \times 5 - 4.9 \times 5^2$, so $h = 24.5$

(b) At the highest point $v = 0$: $0 = 19.6^2 - 2 \times 9.8 \times s$

$s = 19.6$ m above the point of projection

Greatest height above the ground $= 24.5 + 19.6$

Answer: (a) 24.5 m (b) 44.1 m

5 Addition formulae with given sin and cos (All boards)

A is acute and B is obtuse, with $\sin A = \frac{4}{5}$ and $\cos B = -\frac{15}{17}$.

Find the exact values of $\sin(A+B)$ and $\cos(A-B)$.

$\cos A = \frac{3}{5}$ (A acute), $\sin B = \frac{8}{17}$ (B obtuse), so $\tan A = \frac{4}{3}$, $\tan B = -\frac{8}{15}$

$\sin(A+B) = \sin A \cos B + \cos A \sin B = \frac{4}{5} \times (-\frac{15}{17}) + \frac{3}{5} \times \frac{8}{17} = -\frac{36}{85}$

$\cos(A-B) = \cos A \cos B + \sin A \sin B = \frac{3}{5} \times (-\frac{15}{17}) + \frac{4}{5} \times \frac{8}{17} = -\frac{13}{85}$

Answer: $\sin(A+B) = -\frac{36}{85}$, $\cos(A-B) = -\frac{13}{85}$

6 Correcting a recorded value (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

25 values have $\Sigma x = 825$ and $\Sigma x^2 = 27850$.
One value was recorded as 45 but should have been 28.
Find the correct mean and standard deviation (3 s.f.).

$\Sigma x = 825 - 45 + 28 = 808$

$\Sigma x^2 = 27850 - 45^2 + 28^2 = 26609$

Mean $= \frac{808}{25} = 32.3$

sd $= \sqrt{\frac{26609}{25} - (32.3)^2} = 4.45$

Answer: mean 32.3, sd 4.45

7 Volumes of revolution (CIE (9709); Further Maths on other boards)

The region bounded by the curve $y = \sqrt{x}$, the x-axis, $x = 0$ and $x = 2$ is rotated through 360° about the x-axis.
Find the exact volume of the solid formed.

$y^2 = x$, so $V = \pi\int_0^2 y^2\,dx$

$= \pi \times 2$

$V = 2\pi$

Answer: 2π

8 Circle: tangent at a point (AQA, Edexcel, OCR, MEI, Eduqas)

The circle $x^2 + y^2 + 2x - 8y - 8 = 0$ passes through P(3, 7).
(a) Find the equation of the tangent at P, in the form $ax + by = c$.
(b) Find the coordinates of the point where this tangent crosses the x-axis.

Centre $(-1, 4)$. Gradient of radius CP $= \frac{3}{4}$

Gradient of tangent $= -\frac{4}{3}$

$y - 7 = -\frac{4}{3}(x - 3)$: $4x + 3y = 33$

$y = 0$: $x = \frac{33}{4}$

Answer: (a) $4x + 3y = 33$ (b) $(\frac{33}{4}, 0)$

9 Projectile: perpendicular directions (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected from the ground at 28 m s^{-1} at angle α above the horizontal, $\tan\alpha = \frac{12}{5}$. At time T s, while in flight, it is moving perpendicular to its initial direction. Find (a) T, (b) the height of the ball then. Take $g = 9.8$. Give answers to 3 s.f. if not exact.

Initial velocity $(\frac{140}{13}, \frac{336}{13})$. At time T the velocity is $(\frac{140}{13}, \frac{336}{13} - 9.8T)$.

Perpendicular: scalar product = 0: $(\frac{140}{13})^2 + \frac{336}{13}(\frac{336}{13} - 9.8T) = 0$

$28^2 - 9.8T \times \frac{336}{13} = 0$ (using $\sin^2\alpha + \cos^2\alpha = 1$)

(a) $T = 28 \div (9.8 \times \frac{12}{13}) = \frac{65}{21}$

(b) $h = \frac{336}{13} \times \frac{65}{21} - 4.9 \times \frac{65}{21}^2 = \frac{595}{18}$

Answer: (a) T = 3.10 s (b) 33.1 m

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10 Scalar product, magnitude of a sum (CIE (9709); Further Maths on other boards)

$|a| = 8$, $|b| = 2$ and the angle between a and b is 120°.
Find the exact values of $a \cdot b$ and $|a + b|$.

$a \cdot b = |a||b|\cos 120° = 8 \times 2 \times -\frac{1}{2} = -8$

$|a + b|^2 = |a|^2 + 2a \cdot b + |b|^2 = 64 + -16 + 4 = 52$

$|a + b| = 2\sqrt{13}$

Answer: $a \cdot b = -8$, $|a + b| = 2\sqrt{13}$