Day 300 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Arc length and sector area (All boards)

A sector of a circle of radius 9 cm has perimeter 23.4 cm.
Find (a) the angle of the sector in radians, (b) the area of the sector.

Perimeter = $2r + r\theta$: $23.4 = 18 + 9\theta$

$\theta = \frac{5.4}{9} = 0.6$

Area = $\frac{1}{2}r^2\theta = \frac{1}{2} \times 9^2 \times 0.6 = 24.3$

Answer: (a) 0.6 rad (b) 24.3 cm^2

2 Arithmetic sequences: finding a and d (All boards)

The 3rd term of an arithmetic sequence is 11 and the 6th term is 29.
Find the first term and common difference, and the sum of the first 30 terms.

$u_6 - u_3 = 3\,d = 29 - 11$ so $d = 6$

$a + 2\,d = 11$ so $a = -1$

$S_{30} = \frac{30}{2}(2 \times (-1) + 29 \times 6)$

Answer: $a = -1$, $d = 6$, $S_{30} = 2580$

3 Newton's second law with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 5 kg is acted on by forces $F_1 = (23\mathbf{i} - 25\mathbf{j})$ N and F_2. Its acceleration is $(3\mathbf{i} - 2\mathbf{j})$ m s^{-2}. Find (a) F_2, (b) the angle between F_2 and $\mathbf{i}$, to 1 d.p.

$F = ma$: $F_1 + F_2 = 5(3\mathbf{i} - 2\mathbf{j}) = 15\mathbf{i} - 10\mathbf{j}$

(a) $F_2 = (15\mathbf{i} - 10\mathbf{j}) - (23\mathbf{i} - 25\mathbf{j}) = -8\mathbf{i} + 15\mathbf{j}$

(b) $|F_2| = \sqrt{(-8^2 + 15^2)} = 17$

$\cos\theta = -8 \div 17$, so $\theta = 118.1°$

Answer: (a) $F_2 = (-8\mathbf{i} + 15\mathbf{j})$ N (b) 118.1°

4 Double angle equations (All boards)

Solve $\cos 2x - 3\sin x = 1$
for $-180° \le x \le 180°$.

$\cos 2x \equiv 1 - 2\sin^2 x$: $2\sin^2 x + 3\sin x = 0$

Solve the quadratic: $\sin x = 0$ or $\sin x = -\frac{3}{2}$

$\sin x = 0$: $x = -180°, 0°, 180°$

$\sin x = -\frac{3}{2}$ is impossible

Answer: $x = -180°, 0°, 180°$

5 Interpolation for median and percentiles (All boards)

The table summarises the length, l mm, of leaves of 51 items.
Use linear interpolation to estimate the upper quartile.

Position = $0.75 \times 51 = 38.25$

Cumulative frequency before the class $35 \le l < 40$ is 31

Estimate = $35 + \frac{38.25 - 31}{11} \times 5$

$= 38.3$

Answer: 38.3

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6 Factor theorem: roots with surds (All boards)

$f(x) = x^3 + 4x^2 + 2x - 1$.
(a) Show that $(x + 1)$ is a factor of $f(x)$.
(b) Solve $f(x) = 0$, giving surds in simplest form.

$f(-1) = -1 + 4 - 2 - 1 = 0$, so $(x + 1)$ is a factor.

$f(x) = (x + 1)(x^2 + 3x - 1)$

$x^2 + 3x - 1 = 0$ gives $x = \frac{-3 \pm \sqrt{13}}{2}$

Answer: $x = -1$ or $x = \frac{-3 \pm \sqrt{13}}{2}$

7 Bayes' theorem in context (All boards)

5% of people have a disease. The screening test is positive for 96% of those who have it and for 10% of those who do not.
(a) Find the probability that a person tests positive.
(b) Given a positive test, find the probability that the person has the disease (3 s.f.).

(a) $P(+) = 0.05 \times 0.96 + 0.95 \times 0.1$
$= 0.048 + 0.095 = 0.143$

(b) $P(\text{has it} \mid +) = \frac{0.05 \times 0.96}{0.143}$

$= 0.336$

Answer: (a) 0.143 (b) 0.336

8 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $4x^2 + 2xy + y^2 = 108$.
Find the coordinates of the points where the tangent is parallel to the x-axis.

$\frac{dy}{dx} = -\frac{8x + 2y}{2x + 2y}$

Horizontal tangent: $8x + 2y = 0$, so $y = -4x$

Substitute: $12x^2 = 108$, so $x^2 = 9$ and $x = \pm 3$

Points: $(3, -12)$ and $(-3, 12)$

Answer: $(3, -12)$ and $(-3, 12)$

9 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{11x - 5}{(1 - 2x)(1 - 3x)}$.
(a) Express $f(x)$ in partial fractions.
(b) Hence expand $f(x)$ up to x^2.
(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 - 2x)} + \frac{B}{(1 - 3x)}$: $A = -1$, $B = -4$

(b) $-1(1 - 2x)^{-1} = -1(1 + 2x + 4x^2 \ldots)$

Add the series for both fractions.

(c) Need $|2x| < 1$ and $|3x| < 1$

Answer: (a) $\frac{-1}{(1 - 2x)} + \frac{-4}{(1 - 3x)}$ (b) $-5 - 14x - 40x^2$ (c) $|x| < \frac{1}{3}$

10 Connected particles (rough table, string slack) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (6 kg) lies on a rough table, $\mu = 0.1$, joined by a light string over a smooth pulley to particle B (8 kg), 0.8 m above the floor. Released from rest; B lands without rebounding. Find (a) the acceleration of A before B lands, (b) the further distance A moves. Take $g = 9.8$. Give answers to 3 s.f. if not exact.

B: $8g - T = 8a$; A: $T - 0.1 \times 6g = 6a$
Adding: $(8 - 0.1 \times 6)g = 14a$, so $a = 5.18$
Speed of B and A as B lands: $v^2 = 2 \times 5.18 \times 0.8 = 8.288$
After B lands, A decelerates at $\mu g = 0.1 \times 9.8 = 0.98$
$0 = v^2 - 2(\mu g)s$, $s = 8.288 \div 1.96$

Answer: (a) 5.18 m s^{-2} (b) 4.23 m