**Day 30** **A Level Maths: A Tier 10-a-day: Model Solutions**

## 1 Logarithmic equations (All boards)

Solve $\log_2(x + 1) + \log_2(x - 2) = 2$.

Combine: $\log_2[(x + 1)(x - 2)] = 2$, so $(x + 1)(x - 2) = 2^2 = 4$

$x^2 - x - 6 = 0$: $(x - 3)(x + 2) = 0$

$x = 3$ or $x = -2$; $x = -2$ makes $x - 2$ negative, so it is rejected

**Answer: $x = 3$**

## 2 Vertical motion under gravity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is thrown vertically upwards at 39.2 m $s^{-1}$ from a point h m above horizontal ground, and hits the ground 11 s later. Find (a) h, (b) the greatest height above the ground. Take g = 9.8.

(a) Take upwards as positive: $s = -h$, $u = 39.2$, $a = -9.8$, $t = 11$

$-h = 39.2 \times 11 - 4.9 \times 11^2$, so $h = 161.7$

(b) At the highest point $v = 0$: $0 = 39.2^2 - 2 \times 9.8 \times s$

$s = 78.4$ m above the point of projection

Greatest height above the ground = 161.7 + 78.4

**Answer: (a) 161.7 m (b) 240.1 m**

## 3 Surds: sums of fractions (AQA, Edexcel, OCR, MEI, Eduqas)

Show that $\frac{2}{\sqrt{11} - 1} + \frac{4}{\sqrt{11} + 1}$ can be written as $p + q\sqrt{11}$, and find p and q.

Common denominator $(\sqrt{11} - 1)(\sqrt{11} + 1) = 11 - 1 = 10$

Numerator $= 2(\sqrt{11} + 1) + 4(\sqrt{11} - 1)$

$= -2 + 6\sqrt{11}$

Divide by 10.

**Answer: $p = -\frac{1}{5}$, $q = \frac{3}{5}$**

## 4 Beam on two supports (AQA, Edexcel, OCR, MEI)

A uniform beam AB rests on supports at C and D, with a particle attached at B. Find the reactions at C and D. Take g = 9.8.

Moments about C: $R_D \times 3.5 = 10\,g \times 1.5 + 8\,g \times 4.5$

$R_D = 143$

Resolving vertically: $R_C + R_D = 18\,g = 176.4$

$R_C = 176.4 - 143$

**Answer: $R_C = 33.6$ N, $R_D = 142.8$ N**

## 5 Tree diagram with conditional probability (All boards)

The tree diagram shows probabilities for events A and B.

(a) Find P(B′).

(b) Find P(A′ | B′).

(a) $P(B') = 0.7 \times 0.7 + 0.3 \times 0.3 = 0.58$

(b) $P(A' \mid B') = \frac{0.3 \times 0.3}{0.58}$

**Answer: (a) 0.58 (b) $\frac{9}{58}$**

## 6 R-form and solving a sin x + b cos x = c (All boards)

(a) Express 8 cos x − 15 sin x in the form R cos(x + α), where R > 0 and 0° < α < 90°. Give α to 1 d.p.
(b) Solve 8 cos x − 15 sin x = 5 for 0° ≤ x < 360°, giving answers to 1 d.p.

$R^2 = 8^2 + 15^2 = 289$, so R = 17; tan α = $\frac{15}{8}$ gives α = 61.9°

17 cos(x + 61.9°) = 5, so cos(x + 61.9°) = $\frac{5}{17}$

Principal value 72.9°, then use the symmetry of the cosine graph
x = 11.0°, 225.2°
**Answer: (a) R = 17, α = 61.9° (b) x = 11.0°, 225.2°**

## 7 Exponential equations: quadratic in b^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $3 \times 3^{2x} - 4 \times 3^x + 1 = 0$.

Let $y = 3^x$. Then $3y^2 - 4y + 1 = 0$

$3(y - 1)(y - \frac{1}{3}) = 0$

y = 1 or y = $\frac{1}{3}$

$3^x = 1$ gives x = 0; $3^x = \frac{1}{3}$ gives x = −1
**Answer: x = −1 or x = 0**

## 8 Separable differential equations (All boards)

Solve the differential equation

$(1 + x^2)\frac{dy}{dx} = xy$,

given that y = 3 when x = 0 (y > 0). Give y in terms of x.

Separate: $\int \frac{1}{y}\,dy = \int \frac{x}{1 + x^2}\,dx$, so $\ln y = \frac{1}{2}\ln(1 + x^2) + C$

x = 0, y = 3 gives C = ln 3

$y = 3\sqrt{1 + x^2}$
**Answer: $y = 3\sqrt{1 + x^2}$**

## 9 Circle through two points with centre on a line (AQA, Edexcel, OCR, MEI, Eduqas)

A circle passes through A(−4, 13) and B(−4, −7).
Its centre lies on the line y = 3x + 15.
Find the equation of the circle.

Centre (h, 3h + 15) is equidistant from A and B.
$(h + 4)^2 + (3h + 2)^2 = (h + 4)^2 + (3h + 22)^2$
Solving gives h = −4, so centre (−4, 3)
$r^2 = (0)^2 + (10)^2 = 100$
**Answer: $(x + 4)^2 + (y - 3)^2 = 100$**

## 10 Conditional normal probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ N(57, $5^2$). Find P(X > 62 | X > 60) to 3 significant figures.

Since X > 62 implies X > 60: P = $\frac{P(X > 62)}{P(X > 60)}$
P(X > 62) = P(Z > 1.00) = 0.159
P(X > 60) = P(Z > 0.600) = 0.274
P = $\frac{0.159}{0.274}$ = 0.578
**Answer: 0.578**