Day 3 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Lift and cable tension (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lift of mass 400 kg carries a person of mass 75 kg. The cable tension is 4797.5 N and the lift is accelerating. Find (a) the magnitude and direction of the acceleration, (b) the force of the floor on the person. Take g = 9.8.

Whole system (mass 475 kg), taking upwards as positive: $T - 475g = 475a$
$4797.5 - 4655 = 475a$, so $a = 0.3$
(a) a is positive, so the acceleration is 0.3 m s^{-2} upwards
(b) Person: $R - 75g = 75a$, so $R = 75(9.8 + 0.3)$
Answer: (a) 0.3 m s^{-2} upwards (b) 757.5 N

2 Trigonometric equations (quadratic in sin or cos) (All boards)

Solve $2\cos^2\theta - \sin\theta - 2 = 0$
for $0 \leq \theta \leq 2\pi$.

Use $\cos^2\theta \equiv 1 - \sin^2\theta$: $2\sin^2\theta + \sin\theta = 0$
Solving the quadratic: $\sin\theta = -\frac{1}{2}$ or $\sin\theta = 0$
$\sin\theta = -\frac{1}{2}$: $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$
$\sin\theta = 0$: $\theta = 0, \pi, 2\pi$
Answer: $\theta = 0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi$

3 Modulus equations (All boards)

Solve $|x - 8| = |2x + 2|$.

Either $x - 8 = 2x + 2$ or $x - 8 = -(2x + 2)$
First: $-x = 10$
Second: $3x = 6$
Answer: $x = -10$ or $x = 2$

4 Interpolation for median and percentiles (All boards)

The table summarises the journey time, t minutes of 73 items.
Use linear interpolation to estimate the 90th percentile.

Position = $0.9 \times 73 = 65.7$
Cumulative frequency before the class $80 \leq t < 100$ is 46
Estimate = $80 + \frac{65.7 - 46}{20} \times 20$
$= 99.7$
Answer: 99.7

5 Final seconds of a fall (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is dropped from rest from the top of a cliff. In the last 3 seconds before it hits the sea it falls 191.1 m. Find the height of the cliff. Take g = 9.8.

Let the total time of fall be t s. Then $s = 4.9t^2$ for the whole fall.
In the first $(t - 3)$ s the stone falls $4.9(t - 3)^2$.
$4.9t^2 - 4.9(t - 3)^2 = 191.1$
$4.9(6t - 9) = 191.1$, so $t = 8$
Height = $4.9 \times 8^2 = 313.6$
Answer: 313.6 m

6 Differential equations in context (All boards)

Water drains from a tank so that the depth h metres falls at a rate proportional to $\sqrt{h}$. Initially h = 49, and after 6 hours h = 25.

Form and solve a differential equation to find how long, in total, the tank takes to empty.

$\frac{dh}{dt} = -k\sqrt{h}$: $\int h^{-\frac{1}{2}}\, dh = \int -k\, dt$, so $2\sqrt{h} = -kt + C$

t = 0: $C = 2 \times 7 = 14$, so $2\sqrt{h} = 14 - kt$

t = 6, h = 25: $10 = 14 - 6k$, so $k = \frac{2}{3}$

Empty when h = 0: $kt = 14$, so $t = \frac{14}{\frac{2}{3}} = 21$ hours

Answer: 21 hours

7 Regression line from summary data (AQA, Edexcel, OCR, MEI, Eduqas)

n = 9 pairs of (x, y): $\Sigma x = 165$, $\Sigma y = 586$, $\Sigma x^2 = 3465$, $\Sigma xy = 11824$.

(a) Find the equation of the regression line of y on x (3 s.f.).

(b) Estimate y when x = 25.

$S_{xx} = 3465 - \frac{165^2}{9} = 440$, $S_{xy} = 11824 - \frac{165 \times 586}{9} = 1080$

$b = \frac{S_{xy}}{S_{xx}} = 2.46$

$a = \bar{y} - b\,\bar{x} = 65.1 - (2.46)(18.3) = 20.1$

(b) $y = 20.1 + (2.46)(25) = 81.5$

Answer: (a) y = 20.1 + 2.46x (b) 81.5

8 Sum to infinity: two possible ratios (All boards)

The second term of a geometric series is 8 and its sum to infinity is 50.

Find the two possible values of the first term.

$ar = 8$ and $\frac{a}{(1 - r)} = 50$

$a = 50(1 - r)$, so $50(1 - r)\, r = 8$

$50r^2 - 50r + 8 = 0$: $r = \frac{1}{5}$ or $r = \frac{4}{5}$

$a = 50(1 - r) = 40$ or 10

Answer: a = 10 or a = 40

9 Smallest whole number solving an exponential inequality (All boards)

Find the least integer n for which $3^n > 10^7$.

$n \lg 3 > 7$, so $n > \frac{7}{\lg 3} = 14.67$

Since n must be an integer, n = 15

Answer: n = 15

10 Exponential equations: quadratic in b^x (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $3^{2x} - 12 \times 3^x + 27 = 0$.

Let $y = 3^x$. Then $y^2 - 12\,y + 27 = 0$

$(y - 9)(y - 3) = 0$

y = 9 or y = 3

$3^x = 9$ gives x = 2; $3^x = 3$ gives x = 1

Answer: x = 1 or x = 2