Day 299 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Estimating mean and standard deviation (All boards)

The table summarises the mass, m grams, of eggs.
Estimate the mean and standard deviation (3 s.f.).

Midpoints: 5, 15, 25, 35, 45 (n = 51)

$\Sigma fx = 1365$, so mean $= \frac{1365}{51} = 26.8$

$\Sigma fx^2 = 46675$

sd $= \sqrt{\frac{46675}{51} - (26.8)^2} = 14.1$

Answer: mean 26.8, sd 14.1

2 Angle between vectors (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C have coordinates (4, 0, 3), (−3, 1, 2) and (3, 4, −2).
Find angle ABC, in degrees to 1 d.p.

$\text{BA} = \begin{pmatrix} 7 \\ -1 \\ 1 \end{pmatrix}$, $\text{BC} = \begin{pmatrix} 6 \\ 3 \\ -4 \end{pmatrix}$

BA·BC = 35

$|\text{BA}| = \sqrt{51}$, $|\text{BC}| = \sqrt{61}$

cos ABC $= \frac{35}{\sqrt{51} \times \sqrt{61}} = 0.6275$, so angle ABC = 51.1°

Answer: 51.1°

3 Binomial expansion: a coefficient (All boards)

Find the coefficient of x^3 in the expansion of $(3 - x)^8$.

Term in x^3: ${}^8C_3 \times 3^5 \times (-1x)^3$

$= 56 \times 243 \times -1$

Answer: −13608

4 Pulley, string goes slack (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (5 kg) and B (3 kg) hang on a light string over a smooth pulley, A being 2 m above the ground. Released from rest; A lands without rebounding. Find (a) the tension while both move, (b) the greatest height of B above its start. Take g = 9.8.

A: 5 g − T = 5 a; B: T − 3 g = 3 a
a = 2 g ÷ 8 = 2.45, so T = 3(g + a) = 36.75
When A lands, B has risen 2 m with $v^2 = 2 \times 2.45 \times 2 = 9.8$
B then moves freely under gravity: extra height = $v^2 \div (2 \times 9.8) = 0.5$
Total = 2 + 0.5
Answer: (a) 36.75 N (b) 2.5 m

5 Perpendicular bisector and area (All boards)

A is (2, −4) and B is (4, 4).
(a) Find the equation of the perpendicular bisector of AB, in the form ax + by = c.
(b) The perpendicular bisector meets the x-axis at P. Find the area of triangle ABP.

Midpoint of AB = (3, 0); gradient AB = 4

Perpendicular gradient $= -\frac{1}{4}$

Equation: x + 4y = 3
y = 0: P = (3, 0)
Area = ½ |(4 − 2)(0 − (−4)) − (3 − 2)(4 − (−4))|
Answer: (a) x + 4y = 3 (b) 0

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6 Critical values for a binomial distribution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(24, 0.1).

Find the smallest value of c such that P(X ≥ c) < 0.05.

P(X ≥ 6) = 1 − P(X ≤ 5) = 0.0277

P(X ≥ 5) = 1 − P(X ≤ 4) = 0.0851

0.0277 < 0.05 but 0.0851 > 0.05

Answer: c = 6

7 Equation after a sequence of transformations (All boards)

The curve with equation $y = x^2 + 3x + 4$ is transformed by a reflection in the x-axis, followed by a translation of $\begin{pmatrix}3\\1\end{pmatrix}$.

Find the equation of the image curve.

Reflect in x-axis: y becomes −y: $y = -x^2 - 3x - 4$

Translation: replace x by (x − 3) and change y by 1: $y = -x^2 + 3x - 3$

Answer: $y = -x^2 + 3x - 3$

8 R-form and solving a sin x + b cos x = c (All boards)

(a) Express 15 sin θ − 8 cos θ in the form R sin(θ − α), where R > 0 and 0° < α < 90°. Give α to 1 d.p.

(b) Solve 15 sin θ − 8 cos θ = −10 for 0° ≤ θ < 360°, giving answers to 1 d.p.

$R^2 = 15^2 + 8^2 = 289$, so R = 17; tan α = $\frac{8}{15}$ gives α = 28.1°

17 sin(θ − 28.1°) = −10, so sin(θ − 28.1°) = $\frac{-10}{17}$

Principal value −36.0°, then use the symmetry of the sine graph

θ = 244.1°, 352.0°

Answer: (a) R = 17, α = 28.1° (b) θ = 244.1°, 352.0°

9 Ladder in limiting equilibrium (AQA, Edexcel, OCR, MEI)

A uniform ladder AB (5 m, 20 kg) rests with A on rough ground, μ = $\frac{1}{3}$, and B against a smooth wall, B being 3 m above the ground. A man of mass 50 kg climbs the ladder. How far along the ladder from A can he climb before it slips?

Foot of ladder is $\sqrt{(5^2 - 3^2)} = 4$ m from the wall

Vertical: R = (20 + 50)g. Limiting friction: F = μR = $\frac{1}{3}$(20 + 50)g

Horizontal: S = F (wall reaction S)

Moments about A (man at distance x along ladder, horizontal distance x×4/5):

S×3 = 20 g×2 + 50 g×(4 x ÷ 5)

$\frac{70}{3}$×3 = 40 + 40 x (g cancels), so x = 0.75

Answer: 0.75 m

10 Parametric differentiation with trigonometry (All boards)

A curve has parametric equations x = 4(t − sin t), y = 4(1 − cos t).

Find $\frac{dy}{dx}$ in terms of t and the exact equation of the tangent at the point where t = $\frac{2\pi}{3}$, in the form y = mx + c.

$\frac{dy}{dx} = \frac{dy}{dt \div dx/dt} = \frac{\sin t}{1 - \cos t}$

t = $\frac{2\pi}{3}$: gradient m = $\frac{\sqrt{3}}{3}$, point ($\frac{8\pi - 6\sqrt{3}}{3}$, 6)

c = y − mx = $\frac{72 - 8\sqrt{3}\pi}{9}$

Answer: $\frac{dy}{dx} = \frac{\sin t}{1 - \cos t}$; $y = \frac{\sqrt{3}}{3}x + \frac{72 - 8\sqrt{3}\pi}{9}$