Day 298 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A stratified sample of 60 members is taken from three groups.
7 of the sample are from Bronze, which has 105 members.
(a) Find the total number of members.
(b) Silver has 135 members. How many are sampled?

Bronze sampled fraction = $\frac{7}{105} = \frac{1}{15}$

Total = $60 \div \frac{1}{15} = 900$

Silver: $135 \times \frac{1}{15} = 9$

Answer: (a) 900 (b) 9

2 Transformations of a turning point (All boards)

The curve y = f(x) has a minimum point at (−1, −5).
Find the coordinates of the corresponding point on:
(a) y = f(−x)
(b) y = −f(x)
(c) y = f(2x)

(a) y = f(−x): (1, −5)
(b) y = −f(x): (−1, 5)

(c) y = f(2x): $(-\frac{1}{2}, -5)$

A stretch or translation moves the point in the stated direction only.

Answer: (a) (1, −5); (b) (−1, 5); (c) $(-\frac{1}{2}, -5)$

3 Venn diagram with algebra (All boards)

The Venn diagram shows the numbers of 33 students in sets A and B.
(a) Find the value of x.
(b) A student is chosen at random. Find P(A′ | B′).

(a) Total: x + (x + 5) + (x + 4) + 3 = 33
3x + 12 = 33, so x = 7
(b) Regions: 7, 12, 11, 3
Not in B: 10, of which not in A: 3

Answer: (a) x = 7 (b) $\frac{3}{10}$

4 Double angle equations (All boards)

Solve $\cos 2\theta - \sin\theta = -2$
for $0° \leq \theta \leq 360°$.

$\cos 2\theta \equiv 1 - 2\sin^2\theta$: $2\sin^2\theta + \sin\theta - 3 = 0$

Solve the quadratic: $\sin\theta = 1$ or $\sin\theta = -\frac{3}{2}$

$\sin\theta = 1$: $\theta = 90°$

$\sin\theta = -\frac{3}{2}$ is impossible

Answer: $\theta = 90°$

5 Equation of a reciprocal curve from its graph (All boards)

The diagram shows the curve $y = \frac{k}{x - a} + b$ with asymptotes as shown. Find the values of k, a and b, and the coordinates where the curve crosses the axes.

Asymptotes $x = 2$, $y = -2$ so $a = 2$, $b = -2$

Through (4, 1): $1 = \frac{k}{2} - 2$

$k = (1 + 2)(2) = 6$

$x = 0$: $y = -5$; $y = 0$: $x = 5$

Answer: k = 6, a = 2, b = −2; (0, −5) and (5, 0)

6 Foot of perpendicular and distance to a line (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix}3\\0\\1\end{pmatrix} + t\begin{pmatrix}0\\-2\\1\end{pmatrix}$.

Find the coordinates of the foot of the perpendicular from P(1, 3, −3) to l, and the exact distance from P to l.

Foot F = A + t d: $\mathbf{PF} = \begin{pmatrix}2\\-2t-3\\t+4\end{pmatrix}$

$\mathbf{PF}\cdot\mathbf{d} = 0$ gives $5t + 10 = 0$, so $t = -2$

F = (3, 4, −1); $\mathbf{PF} = \begin{pmatrix}2\\1\\2\end{pmatrix}$, so $|\mathbf{PF}| = \sqrt{9} = 3$

Answer: F(3, 4, −1), distance 3

7 Catching up (delayed start) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes P at constant speed 6 m s^{-1}. 8 s later car B leaves P from rest, in the same direction, with acceleration 3 m s^{-2}. B overtakes A at Q. Find (a) the time B takes to reach Q, (b) PQ, (c) the speed of B at Q.

Let B take s seconds; A has then been moving for (s + 8) seconds.

B: $\frac{1}{2}\times 3\times s^2 = 1.5\,s^2$ A: $6(s + 8)$

$1.5\,s^2 - 6\,s - 48 = 0$ so $s = 8$ ($s > 0$)

(b) $PQ = 1.5\times 8^2 = 96$

(c) $v = 0 + at = 3\times 8 = 24$

Answer: (a) 8 s (b) 96 m (c) 24 m s^{-1}

8 Perpendicular vectors and a constant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The vectors $\begin{pmatrix}k\\k\\2\end{pmatrix}$ and $\begin{pmatrix}k\\-3\\-5\end{pmatrix}$ are perpendicular.

Find the possible values of k and, for the larger value, the exact length of the first vector.

Scalar product = 0: $k^2 - 3k - 10 = 0$

$(k + 2)(k - 5) = 0$

$k = -2$ or $k = 5$. For $k = 5$ the first vector is $\begin{pmatrix}5\\5\\2\end{pmatrix}$, with length $\sqrt{54} = 3\sqrt{6}$

Answer: k = −2 or 5; length $3\sqrt{6}$

9 Connected particles on a rough incline (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The system is released from rest and A moves up the rough plane. Find (a) the acceleration, (b) the tension in the light string. Take g = 9.8. Give answers to 3 s.f. if not exact.

sin α = 0.8, cos α = 0.6; friction F = μR = 0.4×5 g cos α

B: 11 g − T = 11 a

A: T − 5 g sin α − F = 5 a

Adding: g(11 − 4 − 1.2) = 16 a

(a) a = 3.5525; (b) T = 11(g − a)

Answer: (a) 3.55 m s^{-2} (b) 68.7 N

10 Range of a composite function (All boards)

$f(x) = x^2 + 6x + 15$, $x \in \mathbb{R}$, and $g(x) = \frac{1}{x+1}$, $x > -1$.

(a) Find the range of f.

(b) Show that gf is defined and find its range.

$f(x) = (x + 3)^2 + 6 \geq 6$

So $f(x) \geq 6 > -1$, which lies in the domain of g, so gf is defined.

$gf(x) = \frac{1}{f(x)+1}$, with $f(x) \geq 6$, so $f(x) + 1 \geq 7$

Largest value is $\frac{1}{7} = \frac{1}{7}$ at the minimum of f; $gf(x) > 0$.

Answer: (a) $f(x) \geq 6$ (b) $0 < gf(x) \leq \frac{1}{7}$