Day 297 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Sum of multiples in a range (All boards)

Find the sum of all the multiples of 12 that lie strictly between 47 and 418.

First multiple 48, last multiple 408, common difference 12.

Number of terms $n = \frac{408 - 48}{12} + 1 = 31$

$S = \frac{31}{2}(48 + 408) = 7068$

Answer: 7068

2 Outliers (AQA, Edexcel, OCR, MEI, Eduqas)

A data set has mean 70 and standard deviation 7.
A value is treated as an outlier if it is more than 2 standard deviations from the mean.
Which of 53, 60, 75, 90 are outliers?

2 standard deviations = 2 × 7 = 14
Lower limit = 70 − 14 = 56
Upper limit = 70 + 14 = 84
Outliers are values below 56 or above 84
Answer: 53, 90

3 Perpendicular vectors and a constant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The vectors $\begin{pmatrix} k \\ k \\ -2 \end{pmatrix}$ and $\begin{pmatrix} k \\ -3 \\ -1 \end{pmatrix}$ are perpendicular.

Find the possible values of k and, for the larger value, the exact length of the first vector.

Scalar product = 0: $k^2 - 3k + 2 = 0$
$(k - 1)(k - 2) = 0$

k = 1 or k = 2. For k = 2 the first vector is $\begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}$, with length $\sqrt{12} = 2\sqrt{3}$

Answer: k = 1 or 2; length $2\sqrt{3}$

4 Sigma notation (All boards)

Evaluate $\sum_{r=7}^{30} (5r + 7)$.

Arithmetic series: first term 42, last term 157, 24 terms

$S = \frac{24}{2}(42 + 157) = 2388$

Answer: 2388

5 Equations with cosec x and cot x (All boards)

Solve $4\text{cosec}^2 x + 12\cot x - 20 = 0$
for $0° < x < 180°$. Give answers to 1 d.p.

Use $\text{cosec}^2 x \equiv 1 + \cot^2 x$: $4\cot^2 x + 12\cot x - 16 = 0$
cot x = 1 or cot x = −4
cot x = 1 means tan x = 1: x = 45.0°
cot x = −4 means $\tan x = -\frac{1}{4}$: x = 166.0°
Answer: x = 45.0°, 166.0°

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6 Circle touching an axis (diagram) (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows a circle that touches the x-axis at A(−3, 0) and passes through B(1, 2). Find the equation of the circle.

Touches the x-axis at A, so the centre is (−3, r) and the radius is r.

B on the circle: $(4)^2 + (2 - r)^2 = r^2$

$16 + 4 - 4r = 0$ so $r = 5$

Answer: $(x + 3)^2 + (y - 5)^2 = 25$

7 Integration by substitution (All boards)

Use the substitution $u = x + 2$ to find the exact value of

$\int_0^{34} \frac{x}{\sqrt{x+2}}\,dx$.

$x = u - 2$, $du = dx$; limits $u = 2$ to $u = 36$

$I = \int_2^{36} \frac{u-2}{\sqrt{u}}\,du = \int_2^{36} (u^{\frac{1}{2}} - 2u^{-\frac{1}{2}})\,du$

$= [\frac{2}{3}u^{\frac{3}{2}} - 4u^{\frac{1}{2}}]$

$= \frac{8\sqrt{2} + 360}{3}$

Answer: $\frac{8\sqrt{2} + 360}{3}$

8 Car and trailer (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 1000 kg car tows a 400 kg trailer by a light tow-bar on a horizontal road. Driving force 1110 N; resistances 300 N (car), 250 N (trailer). Find (a) the acceleration and the tow-bar tension. At 12 m s^{-1} the tow-bar breaks. (b) How far does the trailer then travel?

Whole system: $1110 - 300 - 250 = 1400\,a$, so $a = 0.4$

Trailer: $T - 250 = 400\,a$, so $T = 250 + 400 \times 0.4 = 410$

(b) After the break the trailer decelerates at $250 \div 400 = 0.625$ m s^{-2}

$0 = 12^2 - 2 \times 0.625 \times s$, so $s = 115.2$

Answer: (a) 0.4 m s^{-2}, 410 N (b) 115.2 m

9 Ladder in limiting equilibrium (AQA, Edexcel, OCR, MEI)

A uniform ladder AB (10 m, 30 kg) rests with A on rough ground, $\mu = 0.4$, and B against a smooth wall, B being 8 m above the ground. A man of mass 60 kg climbs the ladder. How far along the ladder from A can he climb before it slips?

Foot of ladder is $\sqrt{(10^2 - 8^2)} = 6$ m from the wall

Vertical: $R = (30 + 60)g$. Limiting friction: $F = \mu R = 0.4(30 + 60)g$

Horizontal: $S = F$ (wall reaction S)

Moments about A (man at distance x along ladder, horizontal distance $x \times 6/10$):

$S \times 8 = 30\,g \times 3 + 60\,g \times (6\,x \div 10)$

$36 \times 8 = 90 + 36\,x$ (g cancels), so $x = 5.5$

Answer: 5.5 m

10 Conditional normal probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(60, 5^2)$. Find $P(X > 71 \mid X > 66)$ to 3 significant figures.

Since $X > 71$ implies $X > 66$: $P = \frac{P(X > 71)}{P(X > 66)}$

$P(X > 71) = P(Z > 2.20) = 0.0139$

$P(X > 66) = P(Z > 1.20) = 0.115$

$P = \frac{0.0139}{0.115} = 0.121$

Answer: 0.121