Day 295 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Venn diagrams and conditional probability (All boards)

P(A) = 0.3, P(B) = 0.75 and P(A ∩ B) = 0.15.
Find
(a) P(A ∪ B)
(b) P(B′ | A′)
(c) P(A′ ∩ B′)

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.75 - 0.15 = 0.9$

(b) $P(B' \mid A') = \frac{P(A' \cap B')}{P(A')} = \frac{1 - 0.9}{1 - 0.3} = \frac{1}{7}$

(c) $P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.9 = 0.1$

Answer: (a) 0.9 (b) $\frac{1}{7}$ (c) 0.1

2 Differentiating a^x (All boards)

$y = 3^x$.
Find the exact gradient of the curve at x = 1 and the equation of the tangent there.

$\frac{dy}{dx} = 1 \times 1 \ln 3 \times 3^x = \ln 3 \times 3^x$

At x = 1: y = 3 and gradient = 3 ln 3
Tangent: $y = (3 \ln 3)x + 3 - 3 \ln 3$

Answer: gradient 3 ln 3; $y = (3 \ln 3)x + 3 - 3 \ln 3$

3 Speed-time graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the speed-time graph for a train journey of 1140 m. Find (a) V, (b) the deceleration in the final stage.

(a) Distance = area of trapezium = ½(80 + 40) × V
1140 = 60 V, so V = 19
(b) Deceleration = V ÷ 25 = 19 ÷ 25

Answer: (a) V = 19 (b) 0.76 m s^{-2}

4 One-tailed binomial hypothesis test (AQA, Edexcel, OCR, MEI, Eduqas)

A claim is that the probability p that a patient recovers is 0.2. In a sample of 28, the number of successes is 3.
Test at the 10% level whether p has decreased.

H_0: p = 0.2, H_1: p < 0.2; X ~ B(28, 0.2)
$P(X \leq 3) = 0.160$
0.160 > 0.1
Do not reject H_0: insufficient evidence of a change in p.

Answer: p-value 0.160; do not reject H_0

5 Arithmetic series: S_n formula (All boards)

The sum of the first n terms of an arithmetic series is $S_n = n^2 - 3n$.
(a) Find the first term and the common difference.
(b) Find u_{20}.

$u_1 = S_1 = -2$
$S_2 = -2$, so $u_2 = 0$
$d = u_2 - u_1 = 2$
$u_{20} = -2 + 19 \times 2$

Answer: (a) a = −2, d = 2 (b) 36

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6 Connected particles (smooth table) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (7 kg) rests on a smooth table, joined by a light string over a smooth pulley at the edge to particle B (3 kg), hanging 0.5 m above the floor. Released from rest. Find (a) the tension, (b) the speed of B as it hits the floor. Take g = 9.8. Give answers to 3 s.f. if not exact.

B: $3g - T = 3a$
A: $T = 7a$
Adding: $3g = 10a$, so $a = 2.94$
(a) $T = 7 \times 2.94 = 20.58$
(b) $v^2 = 0 + 2 \times 2.94 \times 0.5 = 2.94$
Answer: (a) 20.58 N (b) 1.71 m s^{-1}

7 Binomial expansion: (a + bx)^n for rational n (All boards)

Expand $(8 + 3x)^{-\frac{1}{3}}$ in ascending powers of x, up to and including x^2.
State the range of x for which the expansion is valid.

$(8 + 3x)^{-\frac{1}{3}} = 8^{-\frac{1}{3}}(1 + \frac{3x}{8})^{-\frac{1}{3}}$

$8^{-\frac{1}{3}} = \frac{1}{2}$

Expand $(1 + y)^{-\frac{1}{3}}$ with $y = \frac{3x}{8}$

Valid for $|\frac{3x}{8}| < 1$, i.e. $|x| < \frac{8}{3}$

Answer: $\frac{1}{2} - \frac{1}{16}x + \frac{1}{64}x^2$; $|x| < \frac{8}{3}$

8 Intersecting or skew lines (CIE (9709); Further Maths on other boards)

l_1: $\mathbf{r} = \begin{pmatrix}3\\0\\9\end{pmatrix} + \lambda\begin{pmatrix}-1\\1\\2\end{pmatrix}$

l_2: $\mathbf{r} = \begin{pmatrix}4\\-3\\4\end{pmatrix} + \mu\begin{pmatrix}1\\1\\1\end{pmatrix}$

Determine whether l_1 and l_2 intersect. If they do, find the point of intersection.

Equate components: $3 - 1\lambda = 4 + 1\mu$; $0 + 1\lambda = -3 + 1\mu$; $9 + 2\lambda = 4 + 1\mu$
Solving: $\lambda = -2$, $\mu = 1$, and all three equations are satisfied
Point of intersection = (5, −2, 5)
Answer: Intersect at (5, −2, 5)

9 Differentiating sec, cosec, tan and cot (All boards)

$y = \cot(2x)$.
Find $\frac{dy}{dx}$ and its exact value at $x = \frac{\pi}{8}$.

$\frac{dy}{dx} = -2\text{cosec}^2(2x)$

At $x = \frac{\pi}{8}$ the angle $2x = \frac{\pi}{4}$, so the gradient is -4

Answer: $\frac{dy}{dx} = -2\text{cosec}^2(2x)$; -4

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10 Partial fractions: improper fraction (All boards)

Express $\frac{x^2 + 10x - 6}{(x - 2)(x + 4)}$ in the form $1 + \frac{A}{(x - 2)} + \frac{B}{(x + 4)}$, where A and B are constants to be found.

Divide: $x^2 + 10x - 6 = (x^2 + 2x - 8) + (8x + 2)$

So the fraction is $1 + \frac{8x + 2}{(x - 2)(x + 4)}$

$A(x + 4) + B(x - 2) \equiv 8x + 2$

$x = 2$ gives $A = 3$; $x = -4$ gives $B = 5$

Answer: A = 3, B = 5