Day 294 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Forces in equilibrium with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

Forces $F_1 = (-\mathbf{i} + 7\mathbf{j})$ N, $F_2 = (6\mathbf{i} - 8\mathbf{j})$ N and $F_3 = (a\mathbf{i} + b\mathbf{j})$ N keep a 3 kg particle in equilibrium. Find (a) a and b. F_2 is then removed; (b) find the magnitude of the acceleration. Give answers to 3 s.f. if not exact.

(a) Equilibrium: $F_1 + F_2 + F_3 = 0$

$\mathbf{i}$: $-1 + 6 + a = 0$, so $a = -5$

$\mathbf{j}$: $7 + (-8) + b = 0$, so $b = 1$

(b) The resultant is now $F_1 + F_3 = -F_2$, of magnitude $\sqrt{(6^2 + -8^2)} = 10$ N

$a = 10 \div 3$

Answer: (a) $a = -5$, $b = 1$ (b) 3.33 m s^{-2}

2 Circle and line: tangent condition (AQA, Edexcel, OCR, MEI, Eduqas)

The line $12x + 5y = k$ is a tangent to the circle $(x + 3)^2 + (y - 4)^2 = 4$.

Find the two possible values of k.

Centre $(-3, 4)$, radius 2

Distance from centre to the line = radius:

$$\frac{|-16 - k|}{13} = 2$$

$|-16 - k| = 26$

Answer: $k = -42$ or $k = 10$

3 Differentiating a^x (All boards)

$y = 5^{2x}$.

Find the exact gradient of the curve at $x = 0$ and the equation of the tangent there.

$$\frac{dy}{dx} = 1 \times 2 \ln 5 \times 5^{2x} = 2 \ln 5 \times 5^{2x}$$

At $x = 0$: $y = 1$ and gradient $= 2 \ln 5$

Tangent: $y = (2 \ln 5)x + 1$

Answer: gradient $2 \ln 5$; $y = (2 \ln 5)x + 1$

4 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{6} - 2) = -4 - 4\sqrt{6}$, writing x in the form $p + q\sqrt{6}$.

$$x = \frac{-4 - 4\sqrt{6}}{\sqrt{6} - 2}$$

Multiply top and bottom by $\sqrt{6} + 2$.

Numerator: $(-4 - 4\sqrt{6})(\sqrt{6} + 2) = -32 - 12\sqrt{6}$

Denominator: $6 - 4 = 2$

Answer: $x = -16 - 6\sqrt{6}$

5 Trapezium rule with over or under estimate (AQA, Edexcel, OCR, MEI, Eduqas)

Use the trapezium rule with 4 strips to estimate $\int_1^5 2(\sqrt{x})\,dx$, giving your answer to 3 s.f.

State, with a reason, whether your estimate is an overestimate or an underestimate.

h = 1; y-values at x = 1, 2, 3, 4, 5: 2.0000, 2.8284, 3.4641, 4.0000, 4.4721

$$T = \frac{1}{2}[2.0000 + 4.4721 + 2(2.8284 + 3.4641 + 4.0000)]$$

T = 13.5

The curve is concave ($y'' < 0$) on this interval, so the chords lie below the curve: underestimate

Answer: 13.5, an underestimate

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6 One-tailed binomial hypothesis test (AQA, Edexcel, OCR, MEI, Eduqas)

A claim is that the probability p that a seed germinates is 0.3. In a sample of 20, the number of successes is 5. Test at the 1% level whether p has decreased.

H_0: p = 0.3, H_1: p < 0.3; X ~ B(20, 0.3)
P(X ≤ 5) = 0.416
0.416 > 0.01
Do not reject H_0: insufficient evidence of a change in p.

Answer: p-value 0.416; do not reject H_0

7 Reducing a relationship to a straight line (graph) (All boards)

The diagram shows the graph of ln y against x for $y = A e^{kx}$.
Find the exact values of A and k.

ln y = ln A + kx, a straight line with gradient k and intercept ln A
Intercept ln A = 3, so $A = e^3$
Gradient $k = \frac{7 - 3}{4 - 0} = 1$

Answer: $A = e^3$, k = 1

8 Bayes' theorem in context (All boards)

3% of people have a disease. The screening test is positive for 80% of those who have it and for 1% of those who do not.
(a) Find the probability that a person tests positive.
(b) Given a positive test, find the probability that the person has the disease (3 s.f.).

(a) P(+) = 0.03 × 0.8 + 0.97 × 0.01
= 0.024 + 0.0097 = 0.0337
(b) $P(\text{has it} \mid +) = \frac{0.03 \times 0.8}{0.0337}$
= 0.712

Answer: (a) 0.0337 (b) 0.712

9 Acceleration function of time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves on the x-axis. At time t s its acceleration is $(12t - 12)$ m s^{-2}. When t = 0 it is at O with velocity -2 m s^{-1}. Find (a) its velocity when the acceleration is zero, (b) its displacement from O then.

a = 0 when t = 1
$v = \int a\,dt = 6t^2 - 12t - 2$ (using v = −2 when t = 0)
v(1) = −8
$x = \int v\,dt = 2t^3 - 6t^2 - 2t$ (x = 0 when t = 0)
x(1) = −6

Answer: (a) -8 m s^{-1} (b) -6 m

10 Tangent conditions (All boards)

Find the set of values of k for which the line $y = -4x + k$ does not meet the curve $y = x^2 + 2x - 3$.

$x^2 + 2x - 3 = -4x + k$ gives $x^2 + 6x + (-3 - k) = 0$
Discriminant: $6^2 - 4 \times 1(-3 - k) = 48 + 4k$
No intersection: discriminant < 0

Answer: k < -12