Day 292 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Hypothesis test for a Poisson mean (CIE)

Complaints occur as a Poisson process at a mean rate of 2 per day.
After a change, the number recorded in one day is 1.
Test at the 5% level whether the mean rate has decreased.

H_0: $\lambda = 2$, H_1: $\lambda < 2$; $X \sim Po(2)$
$P(X \leq 1) = 0.406$
$0.406 > 0.05$
Do not reject H_0: insufficient evidence of a change.
Answer: p-value 0.406; do not reject H_0

2 Equation of a cubic from its graph (All boards)

The diagram shows a sketch of the curve $y = f(x)$, where $f(x)$ is a cubic.
Find $f(x)$, giving your answer in the form $k(x - p)(x - q)^2$.

Crosses at $x = -4$ and touches at $x = 1$, so $f(x) = k(x + 4)(x - 1)^2$
$f(0) = 4$: $k(4)(-1)^2 = 4$
$k = 1$
Answer: $f(x) = (x + 4)(x - 1)^2$

3 Conservation of momentum (CIE)

Particles A (7 kg) and B (3 kg) move towards each other on a smooth horizontal line at 6 m s^{-1} and 4 m s^{-1}. They collide and coalesce. Find (a) the speed of the combined particle, (b) the kinetic energy lost in the collision.

Conservation of momentum (A's direction positive):
$7\times6 - 3\times4 = 10v$, so $v = 3$
KE before = $\frac{1}{2}\times7\times6^2 + \frac{1}{2}\times3\times4^2 = 150$
KE after = $\frac{1}{2}\times10\times3^2 = 45$
Loss = $150 - 45$
Answer: (a) 3 m s^{-1} (b) 105 J

4 Parametric equations: points on axes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 3t^2 - 1$, $y = t(t^2 - 9)$.
Find the coordinates of the points where the curve meets the x-axis.

$y = 0$: $t = 0$ or $t^2 = 9$, so $t = 0, 3, -3$
$t = 0$: $x = -1$
$t = \pm3$: $x = 26$
Answer: $(-1, 0)$ and $(26, 0)$

5 Bayes' theorem in context (All boards)

0.5% of people have a fault. The inspection test is positive for 90% of those who have it and for 1% of those who do not.
(a) Find the probability that a person tests positive.
(b) Given a positive test, find the probability that the person has the fault (3 s.f.).

(a) $P(+) = 0.005 \times 0.9 + 0.995 \times 0.01$
$= 0.0045 + 0.00995 = 0.01445$

(b) $P(\text{has it} \mid +) = \dfrac{0.005 \times 0.9}{0.01445}$

$= 0.311$
Answer: (a) 0.01445 (b) 0.311

6 Implicit differentiation: tangent to a curve (All boards)

A curve has equation $x^2 - 3xy + 2x = 24$.
Show that P(3, −1) lies on the curve and find the equation of the tangent at P in the form ax + by + c = 0 with integer a, b, c.

Differentiate (product rule on the xy term): $(2x - 3y + 2) + (-3x)\frac{dy}{dx} = 0$

At P(3, −1): $11 + (-9)\frac{dy}{dx} = 0$, so $\frac{dy}{dx} = \frac{11}{9}$

Tangent: $y + 1 = \frac{11}{9}(x - 3)$, i.e. $11x - 9y - 42 = 0$

Answer: $11x - 9y - 42 = 0$

7 Circle touching an axis (diagram) (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows a circle that touches the x-axis at A(2, 0) and passes through B(6, 2).
Find the equation of the circle.

Touches the x-axis at A, so the centre is (2, r) and the radius is r.
B on the circle: $(4)^2 + (2 - r)^2 = r^2$
16 + 4 − 4r = 0 so r = 5
Answer: $(x - 2)^2 + (y - 5)^2 = 25$

8 Foot of perpendicular and distance to a line (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} + t\begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix}$.
Find the coordinates of the foot of the perpendicular from P(2, 2, 2) to l, and the exact distance from P to l.

Foot F = A + t d: $\mathrm{PF} = \begin{pmatrix} -1 \\ -t \\ -t - 4 \end{pmatrix}$
PF·d = 0 gives 2t + 4 = 0, so t = −2
F = (1, 4, 0); $\mathrm{PF} = \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$, so $|\mathrm{PF}| = \sqrt{9} = 3$
Answer: F(1, 4, 0), distance 3

9 Hinged rod held by a string (AQA, Edexcel, OCR, MEI)

A uniform rod AB (20 kg, 2 m) is smoothly hinged at A to a vertical wall, with a 2 kg particle at B. A light string from B to the wall above A holds the rod horizontal, at angle θ to it, $\tan\theta = \frac{12}{5}$. Find (a) the tension, (b) the magnitude of the hinge force. Take g = 9.8. Give answers to 3 s.f. if not exact.

Moments about A: $T\sin\theta \times 2 = 20g \times 1 + 2g \times 2$
$T\sin\theta = (10 + 2)g = 117.6$, so (a) T = 127
Horizontal component of hinge force: $X = T\cos\theta = 49.0$
Vertical: $Y + T\sin\theta = (20 + 2)g$, so $Y = 20g \div 2 = 98$
(b) Force $= \sqrt{(X^2 + Y^2)} = \sqrt{(49.0^2 + 98^2)}$
Answer: (a) 127.4 N (b) 110 N

Day 292 **A Level Maths: A Tier 10-a-day: Model Solutions**

10 Equations with cosec x and cot x (All boards)

Solve $4\text{cosec}^2 x - 14\cot x + 8 = 0$
for $0° < x < 360°$. Give answers to 1 d.p.

Use $\text{cosec}^2 x \equiv 1 + \cot^2 x$: $4\cot^2 x - 14\cot x + 12 = 0$

$\cot x = \frac{3}{2}$ or $\cot x = 2$

$\cot x = \frac{3}{2}$ means $\tan x = \frac{2}{3}$: $x = 33.7°, 213.7°$

$\cot x = 2$ means $\tan x = \frac{1}{2}$: $x = 26.6°, 206.6°$

Answer: $x = 26.6°, 33.7°, 206.6°, 213.7°$