Day 291 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 6 m s^{-1} and accelerates uniformly. She travels 64 m in the next 8 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $64 = 6\times8 + \frac{1}{2}\times a\times8^2$, so $a = 0.5$
(b) $v = u + at = 6 + 0.5\times8 = 10$
(c) For the midpoint, $s = 32$: $v^2 = u^2 + 2as = 6^2 + 2\times0.5\times32$
$v^2 = 68$
Answer: (a) 0.5 m s^{-2} (b) 10 m s^{-1} (c) 8.25 m s^{-1}

2 Interpreting a regression line (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows data and the regression line $y = 30 + 2x$.
(a) Interpret the gradient.
(b) Estimate y when x = 9 and comment.

(a) On average, y increases by 2 hundred sales for each extra 1 °C of x
(b) $y = 30 + 2 \times 9 = 48$
x = 9 is within the range of the data (2 to 10): interpolation, so reliable
Answer: (a) y increases by 2 hundred sales per °C (b) 48, reliable

3 Beam on two supports (AQA, Edexcel, OCR, MEI)

A uniform beam AB rests on supports at C and D, with a particle attached at B. Find the reactions at C and D. Take g = 9.8.

Moments about C: $R_D\times5 = 10\ g\times2 + 10\ g\times7$
$R_D = 176$
Resolving vertically: $R_C + R_D = 20\ g = 196$
$R_C = 196 - 176$
Answer: R_C = 19.6 N, R_D = 176.4 N

4 Dividing a line in a ratio (2D position vectors) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A and B have position vectors $\begin{pmatrix}4\\-2\end{pmatrix}$ and $\begin{pmatrix}0\\6\end{pmatrix}$. The point P lies on AB with AP : PB = 1 : 3.

Find the position vector of P and the exact distance OP.

$\text{OP} = \text{OA} + \frac{1}{4}\text{AB} = \begin{pmatrix}4\\-2\end{pmatrix} + \frac{1}{4}\begin{pmatrix}-4\\8\end{pmatrix}$

$= \begin{pmatrix}3\\0\end{pmatrix}$

$\text{OP} = \sqrt{3^2 + 0^2} = \sqrt{9} = 3$

Answer: $\begin{pmatrix}\mathbf{3}\\\mathbf{0}\end{pmatrix}$, OP = 3

5 Surds: sums of fractions (AQA, Edexcel, OCR, MEI, Eduqas)

Show that $\frac{1}{\sqrt{7}-1} + \frac{4}{\sqrt{7}+1}$ can be written as $p + q\sqrt{7}$, and find p and q.

Common denominator $(\sqrt{7}-1)(\sqrt{7}+1) = 7 - 1 = 6$
Numerator $= 1(\sqrt{7}+1) + 4(\sqrt{7}-1)$
$= -3 + 5\sqrt{7}$
Divide by 6.
Answer: $p = -\frac{1}{2}$, $q = \frac{5}{6}$

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6 Normal then binomial (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The length of a plank is N(52, 8^2). A plank is classed as 'special' if its length is less than 44.
11 planks are chosen at random. Find the probability that exactly 2 are special (3 s.f.).

P(special) = P(X < 44) = P(Z < −1.00) = 0.159
Y ~ B(11, 0.159)
P(Y = 2) = C(11, 2)$(0.159)^2(0.841)^9$ = 0.292
Answer: 0.292

7 Geometric series: smallest n (All boards)

A salary is £20000 in year 1 and increases by 3% each year.
In which year does the salary first exceed £30000?

Salary in year n = $20000 \times 1.03^{n-1}$
$1.03^{n-1} > \frac{3}{2}$
$n - 1 > \frac{\log \frac{3}{2}}{\log 1.03} = 13.72$
n = 15
Answer: year 15

8 Double angle equations (All boards)

Solve $\cos 2\theta - \cos\theta = 2$
for $0 \le \theta \le 2\pi$.

$\cos 2\theta \equiv 2\cos^2\theta - 1$: $2\cos^2\theta - \cos\theta - 3 = 0$
Solve the quadratic: $\cos\theta = -1$ or $\cos\theta = \frac{3}{2}$
$\cos\theta = -1$: $\theta = \pi$
$\cos\theta = \frac{3}{2}$ is impossible
Answer: $\theta = \pi$

9 Area and perimeter of a segment (exact) (All boards)

The circle has centre O and radius 12 cm. Angle AOB = $\frac{\pi}{6}$.

Find the exact area of the shaded segment.

Sector area = $\frac{1}{2}r^2\theta = 12\pi$; triangle OAB = $\frac{1}{2}r^2\sin\theta = 36$
Segment area = sector − triangle = $12\pi - 36$
Answer: area = $12\pi - 36$ cm^2

10 Binomial expansion: finding n and k (All boards)

In the expansion of $(1 + kx)^n$, where n is a positive integer, the coefficient of x is −14 and the coefficient of x^2 is 84.
Find n and k.

$nk = -14$ and $\frac{n(n-1)}{2}k^2 = 84$
$(n-1)k = \frac{2 \times 84}{-14} = -12$ (dividing the second equation by the first)
$nk - (n-1)k = k = -14 - (-12) = -2$
$n = \frac{-14}{-2} = 7$
Answer: n = 7, k = −2