Day 29	A Level Maths: A Tier 10-a-day
1 The 600 items in a list are numbered 1 to 600. A systematic sample of 30 items is taken, starting with a random item among the first few. The first item selected is number 15. Find the sampling interval and the number of the last item selected.	
2 $f(x) = 2x + 4$ and $g(x) = x^2 - 2$. Solve $gf(x) = 2$.	
3 Find the exact value of $\int_0^{\ln 3} (4e^{2x} + 4e^x)\, dx$.	
4 Find the value of $\sum_{r=1}^{8} 5 \times 4^r$.	
5 $g(x) = \frac{1}{17 + 5\sin x + 12\cos x}$ (a) Write $5\sin x + 12\cos x$ in the form $R\sin(x + \alpha)$, with α acute. (b) Find the greatest and least values of $g(x)$ and the smallest positive x at which the greatest value occurs (1 d.p.).	

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6 The curve y = f(x), where $f(x) = \frac{1}{x+1} + 2$, is translated by $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$.

Write down the equations of the asymptotes of the new curve, and find where it crosses the x-axis.

7 A particle is projected at 6.3 m s^{-1} up a line of greatest slope of a rough plane inclined at α, where $\tan\alpha = \frac{12}{5}$. The coefficient of friction is 0.3. Find (a) the greatest distance it moves up the plane, (b) its speed on returning to the start. Take g = 9.8. Give answers to 3 s.f. if not exact.

8 X ~ B(11, 0.5). Find $P(X = 5 \mid X \geq 3)$ to 3 significant figures.

9 l_1: $\mathbf{r} = \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix}$

l_2: $\mathbf{r} = \begin{pmatrix} -3 \\ -3 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

Determine whether l_1 and l_2 intersect. If they do, find the point of intersection.

10 A uniform ladder AB (5 m, 30 kg) rests with A on rough ground, $\mu = 0.4$, and B against a smooth wall, B being 3 m above the ground. A man of mass 60 kg climbs the ladder. How far along the ladder from A can he climb before it slips?