Day 288 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Selections with combinations (OCR, MEI, Eduqas, CIE)

A group of 7 men and 8 women. A team of 4 is chosen at random.
Find the probability that all the team are of the same sex.

Total teams = C(15, 4) = 1365
Same sex: C(7, 4) + C(8, 4) = 35 + 70 = 105

$P = \frac{105}{1365}$

$= \frac{1}{13}$

Answer: $\frac{1}{13}$

2 Horizontal projection (AQA, Edexcel, OCR, MEI, Eduqas)

A ball is projected horizontally at 24 m s^{-1} from a point 44.1 m above horizontal ground. Find (a) the horizontal distance to the landing point, (b) the speed and direction (angle below the horizontal, 1 d.p.) on landing. Take g = 9.8. Give answers to 3 s.f. if not exact.

Vertical: $44.1 = \frac{1}{2} \times 9.8 \times t^2$, so t = 3
(a) Horizontal distance = 24 × 3
(b) Vertical speed on landing = 9.8×3 = 29.4; horizontal speed = 24
Speed = $\sqrt{(24^2 + 29.4^2)} = 38.0$
$\tan \theta = 29.4 \div 24$, $\theta = 50.8°$

Answer: (a) 72 m (b) 38.0 m s^{-1} at 50.8° below the horizontal

3 Small angle approximations: solving an equation (AQA, Edexcel, OCR, MEI, Eduqas)

θ is small and measured in radians. Use small angle approximations to show that
$3\sin 3\theta + \cos \theta = 5.5$
can be written as $a\theta^2 + b\theta + c = 0$ with integers a, b, c. Hence find the approximate solution with $0 < \theta < 0.8$, to 3 s.f.

$\sin 3\theta \approx 3\theta$, $\cos \theta \approx 1 - \frac{1}{2}\theta^2$

$3 \times 3\theta + 1(1 - \frac{1}{2}\theta^2) = 5.5$

Multiply by 2 and rearrange: $\theta^2 - 18\theta + 9 = 0$

$\theta = \frac{18 \pm \sqrt{288}}{2}$, so $\theta = 0.515$

Answer: a = 1, b = −18, c = 9; $\theta \approx 0.515$

4 Sigma notation (All boards)

Evaluate $\sum_{r=8}^{24} (5r - 5)$.

Arithmetic series: first term 35, last term 115, 17 terms

$S = \frac{17}{2}(35 + 115) = 1275$

Answer: 1275

5 Connected rates of change: cones and bowls (AQA, Edexcel, OCR, MEI, Eduqas)

Water pours into a cone, vertex down, at 20 $cm^3 s^{-1}$. The surface radius is always $\frac{h}{2}$.

Find the rate at which the depth is rising when h = 4 cm.

$V = \frac{1}{3}\pi(h/2)^2 h = \frac{\pi h^3}{12}$, so $\frac{dV}{dh} = \frac{\pi h^2}{4}$

$\frac{dh}{dt} = \frac{dV}{dt \div dV/dh} = \frac{20 \times 4}{\pi h^2}$

h = 4: $\frac{dh}{dt} = \frac{5}{\pi}$ cm s^{-1}

Answer: $\frac{5}{\pi}$ **cm/s**

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6 Three-set Venn diagram counts (All boards)

Of 35 students, 15 study Maths (M), 17 Physics (P) and 12 Chemistry (C).
8 study M and P, 5 study M and C, 5 study P and C, and 3 study all three.
One student is chosen at random. Given that a student does not study Maths, find the probability that they study Physics.

Venn regions: M only 5, P only 7, C only 5,
MP only 5, MC only 2, PC only 2, all three 3, outside all 6
Not Maths: $35 - 15 = 20$
Physics but not Maths: $17 - 8 = 9$
Probability = $\frac{9}{20}$

Answer: $\mathbf{\frac{9}{20}}$

7 Catching up (delayed start) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes P at constant speed 7 m s^{-1}. 14 s later car B leaves P from rest, in the same direction, with acceleration 2 m s^{-2}. B overtakes A at Q. Find (a) the time B takes to reach Q, (b) PQ, (c) the speed of B at Q.

Let B take s seconds; A has then been moving for (s + 14) seconds.
B: $\frac{1}{2}\times 2\times s^2 = 1\ s^2$ A: $7(s + 14)$
$1\ s^2 - 7\ s - 98 = 0$ so $s = 14$ $(s > 0)$
(b) $PQ = 1\times 14^2 = 196$
(c) $v = 0 + at = 2\times 14 = 28$
Answer: (a) 14 s (b) 196 m (c) 28 m s^{-1}

8 Recurrence relations: unknown first term (All boards)

A sequence is defined by $u_1 = k$ and $u_{n+1} = -2u_n + 5$.

Given that $u_3 = 11$, find (a) the value of k, (b) $\sum_{r=1}^{4} u_r$.

$u_2 = -2k + 5$, $u_3 = (-2)(-2k + 5) + 5$
$4k - 5 = 11$ so $k = 4$
Terms: 4, −3, 11, −17
Sum = −5
Answer: (a) k = 4 (b) −5

9 Differential equations in context (All boards)

Water drains from a tank so that the depth h metres falls at a rate proportional to $\sqrt{h}$. Initially h = 25, and after 5 hours h = 1.
Form and solve a differential equation to find how long, in total, the tank takes to empty.

$\frac{dh}{dt} = -k\sqrt{h}$: $\int h^{-\frac{1}{2}}\,dh = \int -k\,dt$, so $2\sqrt{h} = -kt + C$
$t = 0$: $C = 2 \times 5 = 10$, so $2\sqrt{h} = 10 - kt$
$t = 5$, $h = 1$: $2 = 10 - 5k$, so $k = \frac{8}{5}$
Empty when $h = 0$: $kt = 10$, so $t = \frac{10}{\frac{8}{5}} = \frac{25}{4}$ hours

Answer: $\mathbf{\frac{25}{4}}$ **hours**

10 Tangent conditions (All boards)

The line $y = kx + 2$ is a tangent to the curve $y = x^2 - 2x + 11$.
(a) Find the possible values of k.
(b) For the larger value of k, find the coordinates of the point of contact.

Equate: $x^2 - 2x + 11 = kx + 2$

$x^2 + (-2 - k)x + 9 = 0$

Tangent: discriminant = 0: $(-2 - k)^2 = 36$

$-2 - k = \pm 6$ so $k = 4$ or -8

$k = 4$: $x = \frac{k - (-2)}{2} = 3$, $y = 14$

Answer: (a) $k = -8$ or $k = 4$ (b) (3, 14)