Day 287 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A company employs 440 people: 160 Sales, 220 Admin and 60 Production.
A stratified sample of size 22 is taken.
Find the number of people sampled from each group.

Sampling fraction = $\frac{22}{440} = \frac{1}{20}$

Sales: $160 \times \frac{1}{20} = 8$

Admin: $220 \times \frac{1}{20} = 11$

Production: $60 \times \frac{1}{20} = 3$

Answer: Sales 8, Admin 11, Production 3

2 Logarithmic equations (All boards)

Solve $\log_3(x + 4) + \log_3(x - 2) = 3$.

Combine: $\log_3[(x + 4)(x - 2)] = 3$, so $(x + 4)(x - 2) = 3^3 = 27$

$x^2 + 2x - 35 = 0$: $(x - 5)(x + 7) = 0$

$x = 5$ or $x = -7$; $x = -7$ makes $x - 2$ negative, so it is rejected

Answer: $x = 5$

3 Sigma notation (All boards)

Find the value of $\sum_{r=1}^{6} 6 \times 3^r$.

Geometric series: first term 18, ratio 3, 6 terms

$S = \frac{18(3^6 - 1)}{(3 - 1)}$

$= \frac{18 \times 728}{2}$

Answer: 6552

4 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 6 m s^{-1} and accelerates uniformly. She travels 100 m in the next 10 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $100 = 6 \times 10 + \frac{1}{2} \times a \times 10^2$, so $a = 0.8$

(b) $v = u + at = 6 + 0.8 \times 10 = 14$

(c) For the midpoint, $s = 50$: $v^2 = u^2 + 2as = 6^2 + 2 \times 0.8 \times 50$

$v^2 = 116$

Answer: (a) 0.8 m s^{-2} (b) 14 m s^{-1} (c) 10.8 m s^{-1}

5 Conditional probability without replacement (All boards)

A bag has 5 red and 7 blue counters. Two are taken at random without replacement.
Given that the counters are the same colour, find the probability they are both red.

P(both red) = $\frac{5}{12} \times \frac{4}{11} = \frac{5}{33}$

P(both blue) = $\frac{7}{12} \times \frac{6}{11} = \frac{7}{22}$

P(same colour) = $\frac{31}{66}$

P(both red | same) = $\frac{\frac{5}{33}}{\frac{31}{66}}$

$= \frac{10}{31}$

Answer: $\frac{10}{31}$

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6 Acceleration function of time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves on the x-axis. At time t s its acceleration is $(12t - 36)$ m s^{-2}. When t = 0 it is at O with velocity -7 m s^{-1}. Find (a) its velocity when the acceleration is zero, (b) its displacement from O then.

a = 0 when t = 3

$v = \int a\,dt = 6t^2 - 36t - 7$ (using v = −7 when t = 0)

v(3) = −61

$x = \int v\,dt = 2t^3 - 18t^2 - 7t$ (x = 0 when t = 0)

x(3) = −129

Answer: (a) −61 m s^{-1} (b) −129 m

7 Separable differential equations (All boards)

Solve the differential equation

$x\frac{dy}{dx} = 3y$,

given that y = 5 when x = 1 (x > 0, y > 0). Give y in terms of x.

Separate: $\int \frac{1}{y}\,dy = \int \frac{3}{x}\,dx$, so $\ln y = 3 \ln x + C$

x = 1, y = 5 gives $C = \ln 5$

$\ln y = \ln(5x^3)$, so $y = 5x^3$

Answer: $y = 5x^3$

8 Binomial expansion: (a + bx)^n for rational n (All boards)

Expand $(2 + x)^{-2}$ in ascending powers of x, up to and including x^2.

State the range of x for which the expansion is valid.

$(2 + x)^{-2} = 2^{-2}(1 + \frac{1x}{2})^{-2}$

$2^{-2} = \frac{1}{4}$

Expand $(1 + y)^{-2}$ with $y = \frac{1x}{2}$

Valid for $|\frac{1x}{2}| < 1$, i.e. $|x| < 2$

Answer: $\frac{1}{4} - \frac{1}{4}x + \frac{3}{16}x^2$; $|x| < 2$

9 Binomial expansion: product with a bracket (All boards)

Find the coefficient of x^4 in the expansion of

$(1 - 3x)(2 - 2x)^6$.

In $(2 - 2x)^6$: coefficient of x^3 is −1280, of x^4 is 960

Coefficient = 1 × 960 + (−3) × (−1280)

Answer: 4800

10 Connected rates of change: cones and bowls (AQA, Edexcel, OCR, MEI, Eduqas)

Water pours into a cone, vertex down, at 30 cm^{3}s^{-1}. The surface radius is always $\frac{h}{3}$.

Find the rate at which the depth is rising when h = 10 cm.

$V = \frac{1}{3}\pi(h/3)^2 h = \frac{\pi h^3}{27}$, so $\frac{dV}{dh} = \frac{\pi h^2}{9}$

$\frac{dh}{dt} = \frac{dV}{dt \div dV/dh} = \frac{30 \times 9}{\pi h^2}$

h = 10: $\frac{dh}{dt} = \frac{27}{10\pi}$ cm s^{-1}

Answer: $\frac{27}{10\pi}$ cm/s