Day 286 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Combined inequalities (All boards)

(a) Solve $3x + 8 \leq 3$.
(b) Solve $x^2 + 5x > 0$.
(c) Hence find the values of x satisfying both inequalities.

(a) $3x + 8 \leq 3$ so $x \leq -\frac{5}{3}$
(b) $(x + 5)x > 0$: critical values -5 and 0
(b) gives $x < -5$ or $x > 0$
(c) Overlap of the two sets.
Answer: (a) $x \leq -\frac{5}{3}$ (b) $x < -5$ or $x > 0$ (c) $x < -5$

2 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily total rainfall (mm) in the same month are:
Heathrow: mean 9.5, standard deviation 2.7;
Leuchars: mean 11, standard deviation 2.3.
Make two comparisons.

Means: Leuchars has the higher mean (11 against 9.5), so on average the total rainfall is greater at Leuchars
Standard deviations: Heathrow is larger at Heathrow (2.7 against 2.3)
so the readings at Heathrow are more variable and those at Leuchars are more consistent
Answer: Leuchars higher on average; Heathrow more variable

3 Pulled at an angle on rough ground (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 5 kg box is pulled along rough horizontal ground, $\mu = 0.1$, by a rope at angle θ above the horizontal, where $\tan\theta = \frac{4}{3}$.

The tension is 40 N. Find (a) the normal reaction, (b) the acceleration of the box. Take $g = 9.8$.

$\sin\theta = 0.8$, $\cos\theta = 0.6$
(a) Vertical: $R + 40\sin\theta = 5g$, so $R = 17$
(b) Horizontal: $40\cos\theta - \mu R = 5a$
$24 - 0.1\times17 = 5a$
$a = 4.46$
Answer: (a) 17 N (b) 4.46 m s^{-2}

4 Arithmetic series: finding n (All boards)

An arithmetic series has first term 8 and common difference 5.
The sum of the first n terms is 426. Find n.

$\frac{n}{2}(2 \times 8 + (n - 1) \times 5) = 426$
$5n^2 + 11n = 852$
$5n^2 + 11n - 852 = 0$
$(n - 12)(5n + 71) = 0$, so $n = 12$ or $n = -\frac{71}{5}$
n must be a positive integer, so $n = 12$
Answer: 12

5 Equations with sec x and tan x (All boards)

Solve $2\sec^2 x - \tan x - 5 = 0$
for $-180° \leq x \leq 180°$. Give answers to 1 d.p.

Use $\sec^2 x \equiv 1 + \tan^2 x$: $2\tan^2 x - \tan x - 3 = 0$
$\tan x = \frac{3}{2}$ or $\tan x = -1$
$\tan x = \frac{3}{2}$: $x = -123.7°, 56.3°$
$\tan x = -1$: $x = -45.0°, 135.0°$
Answer: $x = -123.7°, -45.0°, 56.3°, 135.0°$

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6 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

For a sample of 11 pairs of values of daily rainfall and ice cream sales, r = 0.82.
The critical value for a one-tailed test at the 1% level is 0.6851.
Test, at this level, whether there is positive correlation between the two variables.

H_0: $\rho = 0$, H_1: $\rho > 0$
Critical value 0.6851
$r = 0.82 > 0.6851$
This is in the critical region, so reject H_0.
There is evidence of positive correlation.
Answer: Reject H_0: evidence of correlation

7 Arithmetic series: S_n formula (All boards)

The sum of the first n terms of an arithmetic series is $S_n = 4n^2 - n$.
(a) Find the first term and the common difference.
(b) Find u_{22}.

$u_1 = S_1 = 3$
$S_2 = 14$, so $u_2 = 11$
$d = u_2 - u_1 = 8$
$u_{22} = 3 + 21 \times 8$
Answer: (a) a = 3, d = 8 (b) 171

8 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $2x^2 - 2xy + 2y^2 = 24$.
Find the coordinates of the points where the tangent is parallel to the x-axis.

$$\frac{dy}{dx} = -\frac{4x - 2y}{-2x + 4y}$$

Horizontal tangent: $4x - 2y = 0$, so $y = 2x$
Substitute: $6x^2 = 24$, so $x^2 = 4$ and $x = \pm 2$
Points: (2, 4) and (−2, −4)
Answer: (2, 4) and (−2, −4)

9 Smooth incline: passing a point twice (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is projected up a line of greatest slope of a smooth plane inclined at α. It passes A, 12.25 m from the start, after 2 s and again after 5 s. Find (a) the speed of projection, (b) α to 1 d.p. Take g = 9.8.

Deceleration $a = g \sin\alpha$ (unknown). $s = ut - \frac{1}{2}at^2$ at both times:
$u\times2 - \frac{1}{2}a\times2^2 = u\times5 - \frac{1}{2}a\times5^2 = 12.25$
Subtracting: $u(5 - 2) = \frac{1}{2}a(25 - 4)$, so $u = \frac{1}{2}a(7)$
Substituting: $\frac{1}{2}a(7)(2) - \frac{1}{2}a\times2^2 = 12.25$, so $\frac{1}{2}a\times10 = 12.25$, $a = 2.45$
(a) $u = 1.225\times7 = 8.575$
(b) $9.8 \sin\alpha = 2.45$, $\sin\alpha = 0.25$, $\alpha = 14.5°$
Answer: (a) 8.575 m s^{-1} (b) 14.5°

10 Connected rates of change: cones and bowls (AQA, Edexcel, OCR, MEI, Eduqas)

A hemispherical bowl of radius 5 cm is filled at 30 $cm^3 s^{-1}$. When the depth is h, $V = \pi(5h^2 - \frac{1}{3}h^3)$.

Find $\frac{dh}{dt}$ when h = 4 cm.

$$\frac{dV}{dh} = \pi(10h - h^2) = \pi h(10 - h)$$

$$\frac{dh}{dt} = \frac{30}{\pi h(10 - h)}$$

$h = 4$: $\frac{dh}{dt} = \frac{30}{\pi \times 4 \times 6} = \frac{5}{4\pi}$ cm s^{-1}

Answer: $\frac{5}{4\pi}$ cm/s