Day 285 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Non-uniform rod and equal tensions (AQA, Edexcel, OCR, MEI)

A non-uniform rod AB (length 6 m, mass 8 kg) has its centre of mass at G, AG = 2.25 m. It hangs horizontally from vertical strings at A and B. A particle of mass m kg is fixed at P, AP = 3.75 m, making the tensions equal. Find (a) m, (b) the tension. Take g = 9.8.

Equal tensions T: 2T = (8 + m)g, so T = ½(8 + m)g
Moments about A: T×6 = 8 g×2.25 + m g×3.75
½(8 + m)×6 = 18 + 3.75 m → 24 + 3 m = 18 + 3.75 m
m (3.75 − 3) = 24 − 18, so m = 8
(b) T = ½(8 + 8)×9.8
Answer: (a) m = 8 (b) 78.4 N

2 Change of sign to locate a root (All boards)

$f(x) = e^x - x - 7$.
Show that the equation f(x) = 0 has a root α in the interval [2, 3].

f(x) is continuous on the interval
f(2) = −1.61 < 0
f(3) = 10.1 > 0
A change of sign, so a root lies in [2, 3]
Answer: Sign change: f(2) = −1.61, f(3) = 10.1

3 Partial fractions: improper fraction (All boards)

Express $\frac{x^2 + 6x - 13}{(x + 1)(x - 2)}$ in the form $1 + \frac{A}{(x + 1)} + \frac{B}{(x - 2)}$, where A and B are constants to be found.

Divide: $x^2 + 6x - 13 = (x^2 - x - 2) + (7x - 11)$
So the fraction is $1 + \frac{7x - 11}{(x + 1)(x - 2)}$
$A(x - 2) + B(x + 1) \equiv 7x - 11$
x = −1 gives A = 6; x = 2 gives B = 1
Answer: A = 6, B = 1

4 Perpendicular vectors and a constant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The vectors $\begin{pmatrix} k \\ k \\ 1 \end{pmatrix}$ and $\begin{pmatrix} k \\ 3 \\ 2 \end{pmatrix}$ are perpendicular.

Find the possible values of k and, for the larger value, the exact length of the first vector.

Scalar product = 0: $k^2 + 3k + 2 = 0$
(k + 1)(k + 2) = 0
k = −2 or k = −1. For k = −1 the first vector is $\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$, with length $\sqrt{3} = \sqrt{3}$
Answer: k = −2 or −1; length $\sqrt{3}$

5 Projectile from a cliff top (AQA, Edexcel, OCR, MEI, Eduqas)

A stone is projected from a cliff top 86.4 m above the sea at 25 m s^{-1}, at angle α above the horizontal, $\tan \alpha = \frac{3}{4}$. Find (a) the time to reach the sea, (b) the horizontal distance travelled, (c) the speed on impact. Take g = 9.8. Give answers to 3 s.f. if not exact.

Horizontal component 20, vertical component 15 (upwards)
(a) $-86.4 = 15\,t - 4.9t^2$, so $4.9t^2 - 15\,t - 86.4 = 0$
t = [15 + √1918.44] ÷ 9.8 = 6 (taking the positive root)
(b) Distance = 20×6
(c) Vertical speed = 15 − 9.8×6 = −43.8; speed = $\sqrt{(20^2 + 43.8^2)}$
Answer: (a) 6 s (b) 120 m (c) 48.2 m s^{-1}

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6 Range of a composite function (All boards)

$f(x) = x^2 + 2x + 8$, $x \in \mathbb{R}$, and $g(x) = \frac{2}{x - 5}$, $x > 5$.

(a) Find the range of f.
(b) Show that gf is defined and find its range.

$f(x) = (x + 1)^2 + 7 \geq 7$
So $f(x) \geq 7 > 5$, which lies in the domain of g, so gf is defined.

$gf(x) = \frac{2}{f(x) - 5}$, with $f(x) \geq 7$, so $f(x) - 5 \geq 2$

Largest value is $\frac{2}{2} = 1$ at the minimum of f; $gf(x) > 0$.

Answer: (a) $f(x) \geq 7$ (b) $0 < gf(x) \leq 1$

7 Normal approximation to the Poisson (CIE)

$X \sim Po(16)$. Use a suitable approximation to find $P(X \leq 24)$, to 3 significant figures.

$\lambda > 15$, so $Y \sim N(16, 16)$ approximately
Continuity correction: $P(Y < 24.5)$
$z = 2.12$, so probability = 0.983
Answer: 0.983

8 Binomial expansion: (a + bx)^n for rational n (All boards)

Expand $(3 - 2x)^{-1}$ in ascending powers of x, up to and including x^2.
State the range of x for which the expansion is valid.

$(3 - 2x)^{-1} = 3^{-1}(1 + \frac{-2x}{3})^{-1}$

$3^{-1} = \frac{1}{3}$

Expand $(1 + y)^{-1}$ with $y = \frac{-2x}{3}$

Valid for $|\frac{-2x}{3}| < 1$, i.e. $|x| < \frac{3}{2}$

Answer: $\frac{1}{3} + \frac{2}{9}x + \frac{4}{27}x^2$; $|x| < \frac{3}{2}$

9 Tangent through the origin (All boards)

The tangent to the curve $y = \ln(3x)$ at the point P passes through the origin.
Find the exact coordinates of P and the equation of the tangent.

$\frac{dy}{dx} = \frac{1}{x}$; tangent at x = a: $y - \ln(3a) = \frac{1}{a}(x - a)$

Through (0, 0): $-\ln(3a) = -1$, so $3a = e$ and $a = \frac{e}{3}$

$P = (\frac{e}{3}, 1)$, gradient $\frac{1}{a} = 3e^{-1}$, so the tangent is $y = 3e^{-1}x$

Answer: $P(\frac{e}{3}, 1)$; $y = 3e^{-1}x$

10 Conditional normal probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(50, 8^2)$. Find $P(X > 59 \mid X > 55)$ to 3 significant figures.

Since $X > 59$ implies $X > 55$: $P = \frac{P(X > 59)}{P(X > 55)}$
$P(X > 59) = P(Z > 1.12) = 0.130$
$P(X > 55) = P(Z > 0.625) = 0.266$
$P = \frac{0.130}{0.266} = 0.490$
Answer: 0.490