Day 284 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Acceleration function of time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves on the x-axis. At time t s its acceleration is $(3t - 6)$ m s^{-2}. When t = 0 it is at O with velocity 4 m s^{-1}. Find (a) its velocity when the acceleration is zero, (b) its displacement from O then.

a = 0 when t = 2

$v = \int a\,dt = 1.5t^2 - 6t + 4$ (using v = 4 when t = 0)

$v(2) = -2$

$x = \int v\,dt = 0.5t^3 - 3t^2 + 4t$ (x = 0 when t = 0)

$x(2) = 0$

Answer: (a) -2 m s^{-1} (b) 0 m

2 Discriminant: range of k (All boards)

The equation $x^2 + (k + 1)x + (2k + 2) = 0$ has no real roots.
Find the set of possible values of k.

Discriminant: $b^2 - 4ac = (k + 1)^2 - 4(2k + 2)$

$= k^2 - 6k - 7$

Roots of the quadratic in k: $k = -1$ and $k = 7$

Required: $b^2 - 4ac < 0$

Answer: $-1 < k < 7$

3 Venn diagrams and conditional probability (All boards)

P(A) = 0.65, P(B) = 0.3 and P(A ∩ B) = 0.05.
Find
(a) P(A ∪ B)
(b) P(A′ ∩ B′)
(c) P(A | B′)

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.65 + 0.3 - 0.05 = 0.9$

(b) $P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.9 = 0.1$

(c) $P(A \mid B') = \frac{P(A \cap B')}{P(B')} = \frac{0.65 - 0.05}{1 - 0.3} = \frac{6}{7}$

Answer: (a) 0.9 (b) 0.1 (c) $\frac{6}{7}$

4 Conservation of momentum (CIE)

Particles A (2 kg) and B (4 kg) move in the same direction on a smooth horizontal line at 6 m s^{-1} and 6 m s^{-1}. They collide and coalesce. Find (a) the speed of the combined particle, (b) the kinetic energy lost in the collision.

Conservation of momentum (A's direction positive):

$2\times6 + 4\times6 = 6v$, so $v = 6$

KE before = $\frac{1}{2}\times2\times6^2 + \frac{1}{2}\times4\times6^2 = 108$

KE after = $\frac{1}{2}\times6\times6^2 = 108$

Loss = $108 - 108$

Answer: (a) 6 m s^{-1} (b) 0 J

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5 Geometric distribution (CIE)

On each independent trial, the probability that a biased die shows a six is $\frac{1}{10}$.

X is the number of trials up to and including the first success.
(a) Find P(X = 5) and E(X).
(b) Find the least m with P(X ≤ m) > 0.9.

(a) $P(X = 5) = (\frac{9}{10})^4 \times \frac{1}{10} = \frac{6561}{100000}$; $E(X) = \frac{1}{p} = 10$

(b) $P(X \leq m) = 1 - (\frac{9}{10})^m > 0.9$

$(\frac{9}{10})^m < 0.1$, so $m > 21.9$

Least integer m = 22

Answer: (a) $\frac{6561}{100000}$, E(X) = 10 (b) m = 22

6 Volumes of revolution (CIE (9709); Further Maths on other boards)

The region bounded by the curve $y = e^{3x}$, the x-axis, x = 0 and x = 2 is rotated through 360° about the x-axis. Find the exact volume of the solid formed.

$y^2 = e^{6x}$, so $V = \pi\int_0^2 y^2\,dx$

$= \pi \times \frac{e^{12} - 1}{6}$

$V = \frac{\pi e^{12} - \pi}{6}$

Answer: $\frac{\pi e^{12} - \pi}{6}$

7 Circle touching an axis (diagram) (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows a circle that touches the x-axis at A(−4, 0) and passes through B(−1, 3). Find the equation of the circle.

Touches the x-axis at A, so the centre is (−4, r) and the radius is r.
B on the circle: $(3)^2 + (3 - r)^2 = r^2$
$9 + 9 - 6r = 0$ so r = 3
Answer: $(x + 4)^2 + (y - 3)^2 = 9$

8 Intersecting or skew lines (CIE (9709); Further Maths on other boards)

l_1: $\mathbf{r} = \begin{pmatrix}4\\6\\1\end{pmatrix} + \lambda\begin{pmatrix}3\\3\\-1\end{pmatrix}$

l_2: $\mathbf{r} = \begin{pmatrix}-2\\0\\3\end{pmatrix} + \mu\begin{pmatrix}-1\\3\\-3\end{pmatrix}$

Determine whether l_1 and l_2 intersect. If they do, find the point of intersection.

Equate components: $4 + 3\lambda = -2 - 1\mu$; $6 + 3\lambda = 0 + 3\mu$; $1 - 1\lambda = 3 - 3\mu$
Solving: $\lambda = -2$, $\mu = 0$, and all three equations are satisfied
Point of intersection = (−2, 0, 3)
Answer: Intersect at (−2, 0, 3)

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9 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{2} - 4) = 5 + 4\sqrt{2}$, writing x in the form $p + q\sqrt{2}$.

$x = \dfrac{5 + 4\sqrt{2}}{\sqrt{2} - 4}$

Multiply top and bottom by $\sqrt{2} + 4$.

Numerator: $(5 + 4\sqrt{2})(\sqrt{2} + 4) = 28 + 21\sqrt{2}$

Denominator: $2 - 16 = -14$

Answer: $x = -2 - \frac{3}{2}\sqrt{2}$

10 Differential equations in context (All boards)

Water drains from a tank so that the depth h metres falls at a rate proportional to $\sqrt{h}$. Initially h = 16, and after 5 hours h = 9.

Form and solve a differential equation to find how long, in total, the tank takes to empty.

$\dfrac{dh}{dt} = -k\sqrt{h}$: $\int h^{-\frac{1}{2}}\, dh = \int -k\, dt$, so $2\sqrt{h} = -kt + C$

$t = 0$: $C = 2 \times 4 = 8$, so $2\sqrt{h} = 8 - kt$

$t = 5$, $h = 9$: $6 = 8 - 5k$, so $k = \dfrac{2}{5}$

Empty when $h = 0$: $kt = 8$, so $t = \dfrac{8}{\frac{2}{5}} = 20$ hours

Answer: 20 hours