Day 283 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Conservation of momentum (CIE)

Particles A (3 kg) and B (5 kg) move towards each other on a smooth horizontal line at 7 m s^{-1} and 3 m s^{-1}. They collide directly, and A then moves in the opposite direction at 3 m s^{-1}. Find (a) the speed of B after the collision, (b) the magnitude of the impulse of A on B.

Take the initial direction of A as positive. Conservation of momentum:
$3\times7 + 5\times(-3) = 3\times(-3) + 5v$
$6 = -9 + 5v$, so $v = 3$
(b) Impulse on B = change in momentum of B $= 5(3 - (-3))$
$= 5 \times 6$

Answer: (a) 3 m s^{-1} (b) 30 N s

2 Laws of logarithms in terms of p and q (All boards)

Given that $\log_a 2 = p$ and $\log_a 3 = q$,

express $\log_a\left(\frac{16}{\sqrt{27}}\right)$ in terms of p and q.

$\frac{16}{\sqrt{27}} = 2^4 \times 3^{-\frac{3}{2}}$

$\log_a(2^4 \times 3^{-\frac{3}{2}}) = 4\log_a 2 - \frac{3}{2}\log_a 3$

$= 4p - \frac{3}{2}q$

Answer: $4p - \frac{3}{2}q$

3 Sum of multiples in a range (All boards)

Find the sum of all the integers from 1 to 139 inclusive that are not multiples of 4.

Sum of 1 to 139 $= \frac{139 \times 140}{2} = 9730$

Multiples of 4 up to 139: 4, 8, ..., 136 (34 terms)

Their sum $= \frac{34}{2}(4 + 136) = 2380$

Required sum $= 9730 - 2380$

Answer: 7350

4 Geometric distribution (CIE)

On each independent trial, the probability that a basketball player scores is $\frac{1}{4}$.

X is the number of trials up to and including the first success.
(a) Find P(X = 3) and E(X).
(b) Find the least m with $P(X \leq m) > 0.9$.

(a) $P(X = 3) = (\frac{3}{4})^2 \times \frac{1}{4} = \frac{9}{64}$; $E(X) = \frac{1}{p} = 4$

(b) $P(X \leq m) = 1 - (\frac{3}{4})^m > 0.9$

$(\frac{3}{4})^m < 0.1$, so $m > 8.00$

Least integer m = 9

Answer: (a) $\frac{9}{64}$, E(X) = 4 (b) m = 9

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5 Connected particles on a rough incline (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The system is released from rest and A moves up the rough plane. Find (a) the acceleration, (b) the tension in the light string. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin\alpha = \frac{12}{13}$, $\cos\alpha = \frac{5}{13}$; friction $F = \mu R = 0.25 \times 5\,g\cos\alpha$

B: $9g - T = 9a$

A: $T - 5g\sin\alpha - F = 5a$

Adding: $g(9 - \frac{60}{13} - \frac{25}{52}) = 14a$

(a) $a = 2.73$; (b) $T = 9(g - a)$

Answer: (a) 2.73 m s^{-2} (b) 63.6 N

6 Arithmetic series: finding n (All boards)

An arithmetic series has first term 13 and common difference 1.
The sum of the first n terms is 247. Find n.

$\frac{n}{2}(2 \times 13 + (n - 1) \times 1) = 247$

$n^2 + 25n = 494$

$n^2 + 25n - 494 = 0$

$(n - 13)(n + 38) = 0$, so $n = 13$ or $n = -38$

n must be a positive integer, so $n = 13$

Answer: 13

7 Conditional normal probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(58, 10^2)$. Find $P(X > 68 \mid X > 63)$ to 3 significant figures.

Since $X > 68$ implies $X > 63$: $P = \frac{P(X > 68)}{P(X > 63)}$

$P(X > 68) = P(Z > 1.00) = 0.159$

$P(X > 63) = P(Z > 0.500) = 0.309$

$P = \frac{0.159}{0.309} = 0.514$

Answer: 0.514

8 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $x^2 - xy + 2y^2 = 28$.
Find the coordinates of the points where the tangent is parallel to the x-axis.

$\frac{dy}{dx} = -\frac{2x - y}{-x + 4y}$

Horizontal tangent: $2x - y = 0$, so $y = 2x$

Substitute: $7x^2 = 28$, so $x^2 = 4$ and $x = \pm 2$

Points: (2, 4) and (−2, −4)

Answer: (2, 4) and (−2, −4)

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9 Geometric series: smallest n (All boards)

A geometric series has first term 3 and common ratio $\frac{6}{5}$.

Find the smallest n for which the sum of the first n terms exceeds 1000.

$S_n = \frac{3(\frac{6}{5}^n - 1)}{(\frac{6}{5} - 1)} > 1000$

$\frac{6}{5}^n > \frac{203}{3}$

$n > \frac{\log \frac{203}{3}}{\log \frac{6}{5}} = 23.12$

Check: $S_{23} = 978.7$, $S_{24} = 1177.5$

Answer: n = 24

10 Tangent through the origin (All boards)

The tangent to the curve $y = e^{\frac{1}{3}x}$ at the point P passes through the origin.
Find the coordinates of P and the equation of the tangent.

$\frac{dy}{dx} = \frac{1}{3}e^{\frac{1}{3}x}$; at x = a the tangent is $y - e^{\frac{1}{3}a} = \frac{1}{3}e^{\frac{1}{3}a}(x - a)$

Through (0, 0): $-e^{\frac{1}{3}a} = -\frac{1}{3}a\, e^{\frac{1}{3}a}$, so $\frac{1}{3}a = 1$ and a = 3

P = (3, e) and gradient ke = $\frac{1}{3}e$

Answer: P(3, e); y = $\frac{1}{3}$ex