Day 282 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Venn diagrams and conditional probability (All boards)

P(A) = 0.3, P(B) = 0.8 and P(A ∩ B) = 0.15.
Find
(a) P(A ∪ B)
(b) P(A′ ∩ B′)
(c) P(A | B)

(a) P(A ∪ B) = P(A) + P(B) − P(A ∩ B) = 0.3 + 0.8 − 0.15 = 0.95
(b) P(A′ ∩ B′) = 1 − P(A ∪ B) = 1 − 0.95 = 0.05
(c) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.8} = \frac{3}{16}$

Answer: (a) 0.95 (b) 0.05 (c) $\frac{3}{16}$

2 Arithmetic sequences: finding a and d (All boards)

The 6th term of an arithmetic sequence is −29 and the 12th term is −59.
Find the first term and common difference, and the sum of the first 25 terms.

$u_{12} - u_6 = 6d = -59 - (-29)$ so $d = -5$
$a + 5d = -29$ so $a = -4$
$S_{25} = \frac{25}{2}(2 \times (-4) + 24 \times (-5))$

Answer: a = −4, d = −5, S_{25} = −1600

3 Ladder in limiting equilibrium (AQA, Edexcel, OCR, MEI)

A uniform ladder AB (25 m, 20 kg) rests with A on rough ground, μ = 0.2, and B against a smooth wall, B being 24 m above the ground. A man of mass 50 kg climbs the ladder. How far along the ladder from A can he climb before it slips?

Foot of ladder is $\sqrt{(25^2 - 24^2)} = 7$ m from the wall
Vertical: R = (20 + 50)g. Limiting friction: F = μR = 0.2(20 + 50)g
Horizontal: S = F (wall reaction S)
Moments about A (man at distance x along ladder, horizontal distance x×7/25):
S×24 = 20 g×3.5 + 50 g×(7 x ÷ 25)
14×24 = 70 + 14 x (g cancels), so x = 19

Answer: 19 m

4 Exponential equations reducing to quadratics (All boards)

Solve $4^x - 32^x - 4 = 0$.
Give your answer to 3 significant figures if not exact.

$4^x = (2^x)^2$. Let $u = 2^x$: $u^2 - 3u - 4 = 0$
(u − 4)(u + 1) = 0, so u = 4 or u = −1
2^x is always positive, so reject u = −1; $2^x = 4$ gives x = 2

Answer: x = 2

5 Connected particles (rough table, string slack) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (3 kg) lies on a rough table, μ = 0.5, joined by a light string over a smooth pulley to particle B (5 kg), 2 m above the floor. Released from rest; B lands without rebounding. Find (a) the acceleration of A before B lands, (b) the further distance A moves. Take g = 9.8. Give answers to 3 s.f. if not exact.

B: 5 g − T = 5 a; A: T − 0.5×3 g = 3 a
Adding: (5 − 0.5×3)g = 8 a, so a = 4.2875
Speed of B and A as B lands: $v^2 = 2 \times 4.2875 \times 2 = 17.15$
After B lands, A decelerates at μg = 0.5×9.8 = 4.9
$0 = v^2 - 2(\mu g)s$, s = 17.15 ÷ 9.8

Answer: (a) 4.29 m s^{-2} (b) 1.75 m

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6 Correcting a recorded value (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

12 values have $\Sigma x = 672$ and $\Sigma x^2 = 37932$.
One value was recorded as 47 but should have been 51.
Find the correct mean and standard deviation (3 s.f.).

$\Sigma x = 672 - 47 + 51 = 676$

$\Sigma x^2 = 37932 - 47^2 + 51^2 = 38324$

Mean $= \frac{676}{12} = 56.3$

sd $= \sqrt{\frac{38324}{12} - (56.3)^2} = 4.50$

Answer: mean 56.3, sd 4.50

7 Inverse trigonometric functions (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = 3 + 2\arccos(3x)$.

Find the exact value of x for which $f(x) = \frac{\pi + 6}{2}$.

Hence state the range of f.

$2\arccos(3x) = \frac{\pi}{2}$, so $\arccos(3x) = \frac{\pi}{4}$

$3x = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$, so $x = \frac{\sqrt{2}}{6}$

arccos takes values from 0 to π, so f(x) runs from 3 to $2\pi + 3$

Answer: $x = \frac{\sqrt{2}}{6}$; range $3 \leq f(x) \leq 2\pi + 3$

8 Surds: sums of fractions (AQA, Edexcel, OCR, MEI, Eduqas)

Show that $\frac{2}{\sqrt{2} - 1} + \frac{3}{\sqrt{2} + 1}$ can be written as $p + q\sqrt{2}$, and find p and q.

Common denominator $(\sqrt{2} - 1)(\sqrt{2} + 1) = 2 - 1 = 1$

Numerator $= 2(\sqrt{2} + 1) + 3(\sqrt{2} - 1)$

$= -1 + 5\sqrt{2}$

Divide by 1.

Answer: $p = -1$, $q = 5$

9 Circle and line: chord length (AQA, Edexcel, OCR, MEI, Eduqas)

The line $12x + 5y = -49$ meets the circle $(x + 1)^2 + (y + 4)^2 = 9$ at the points A and B. Find the exact length of AB.

Centre $(-1, -4)$, radius 3. Distance from centre to line:

$d = \frac{|17|}{13} = \frac{17}{13}$

Half-chord$^2 = 9 - (\frac{17}{13})^2 = \frac{1232}{169}$

$AB = 2\sqrt{\frac{1232}{169}}$

Answer: $\frac{104\sqrt{77}}{169}$

10 Product rule: stationary points (All boards)

$y = (x + 2)e^{-x}$.

Find the exact coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = 1\,e^{-x} - (x + 2)e^{-x} = (-x - 1)e^{-x}$

$e^{-x} > 0$, so $\frac{dy}{dx} = 0$ gives $-x - 1 = 0$ and $x = -1$

$y = e$; $\frac{d^2y}{dx^2}$ is negative there, so it is a maximum

Answer: $(-1, e)$, maximum