Day 280 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 3 m s^{-1} and accelerates uniformly. She travels 43.2 m in the next 8 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $43.2 = 3\times8 + \frac{1}{2}\times a\times8^2$, so $a = 0.6$
(b) $v = u + at = 3 + 0.6\times8 = 7.8$
(c) For the midpoint, $s = 21.6$: $v^2 = u^2 + 2as = 3^2 + 2\times0.6\times21.6$
$v^2 = 34.92$
Answer: (a) 0.6 m s^{-2} (b) 7.8 m s^{-1} (c) 5.91 m s^{-1}

2 Index equations solved with logarithms (All boards)

Solve $7^{4x-3} = 2^{3x-3}$.
Give your answer to 3 significant figures.

Take logs: $(4x - 3)\ln 7 = (3x - 3)\ln 2$
Collect x terms: $x(4\ln 7 - 3\ln 2) = -3\ln 2 + 3\ln 7$
$x = \frac{-3\ln 2 + 3\ln 7}{4\ln 7 - 3\ln 2}$
$x = 0.659$
Answer: x = 0.659

3 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

The 296 items in a list are numbered 1 to 296.
A systematic sample of 37 items is taken, starting with a random item among the first few.
The first item selected is number 2.
Find the sampling interval and the number of the last item selected.

Interval = 296 ÷ 37 = 8
Items selected: 2, 10, 18, ...
Last = 2 + (37 − 1) × 8 = 290
Answer: Interval 8, last item 290

4 Connected rates of change: spheres, circles and cubes (AQA, Edexcel, OCR, MEI, Eduqas)

The volume of a spherical balloon increases at a constant rate of 100 cm^3s^{-1}.
Find the rates of increase of its radius and of its surface area when r = 10 cm.

$V = \frac{4}{3}\pi r^3$, so $\frac{dV}{dr} = 4\pi r^2$. $\frac{dr}{dt} = \frac{dV}{dt \div dV/dr} = \frac{100}{4\pi r^2}$

r = 10: $\frac{dr}{dt} = \frac{1}{4\pi}$ cm s^{-1}

$A = 4\pi r^2$, so $\frac{dA}{dr} = 8\pi r$ and $\frac{dA}{dt} = 8\pi \times 10 \times \frac{1}{4\pi} = 20$ cm^2s^{-1}

Answer: $\frac{dr}{dt} = \frac{1}{4\pi}$ cm/s, $\frac{dA}{dt} = 20$ cm^2/s

5 Recurrence relations: unknown first term (All boards)

A sequence is defined by $u_1 = k$ and $u_{n+1} = -u_n + 6$.

Given that $u_3 = -3$, find (a) the value of k, (b) $\sum_{r=1}^{4} u_r$.

$u_2 = -1\,k + 6$, $u_3 = (-1)(-1\,k + 6) + 6$
$k = -3$ so $k = -3$
Terms: −3, 9, −3, 9
Sum = 12
Answer: (a) k = −3 (b) 12

6 Exponential growth and decay modelling (All boards)

A car costs £18000 new. Its value falls by 8% each year, so after t years its value is £V.
(a) Write down V in terms of t. (b) Find, to 3 s.f., the time when the value is £6000.

$V = 18000 \times 0.92^t$

$18000 \times 0.92^t = 6000$ gives $0.92^t = \frac{1}{3}$

$t = \frac{\ln(1/3)}{\ln 0.92} = 13.2$ years

Answer: (a) $V = 18000 \times 0.92^t$ (b) $t = 13.2$ years

7 Bayes' theorem in context (All boards)

5% of people have a fault. The inspection test is positive for 95% of those who have it and for 10% of those who do not.
(a) Find the probability that a person tests positive.
(b) Given a positive test, find the probability that the person has the fault (3 s.f.).

(a) $P(+) = 0.05 \times 0.95 + 0.95 \times 0.1$
$= 0.0475 + 0.095 = 0.1425$

(b) $P(\text{has it} \mid +) = \frac{0.05 \times 0.95}{0.1425}$

$= 0.333$

Answer: (a) 0.1425 (b) 0.333

8 Circle and line: chord length (AQA, Edexcel, OCR, MEI, Eduqas)

The line $4x - 3y = 1$ meets the circle $(x - 2)^2 + (y + 2)^2 = 36$ at the points A and B. Find the exact length of AB.

Centre $(2, -2)$, radius 6. Distance from centre to line:

$d = \frac{|13|}{5} = \frac{13}{5}$

$\text{Half-chord}^2 = 36 - (\frac{13}{5})^2 = \frac{731}{25}$

$AB = 2\sqrt{\frac{731}{25}}$

Answer: $\frac{10\sqrt{731}}{25}$

9 Projected up a rough incline (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is projected at 9.8 m s^{-1} up a line of greatest slope of a rough plane inclined at α, where $\tan\alpha = \frac{5}{12}$. The coefficient of friction is 0.4. Find (a) the greatest distance it moves up the plane, (b) its speed on returning to the start. Take $g = 9.8$.

Up: deceleration $= g(\sin\alpha + \mu\cos\alpha) = 9.8(\frac{5}{13} + 0.4 \times \frac{12}{13}) = \frac{2401}{325}$

(a) $0 = 9.8^2 - 2 \times \frac{2401}{325} \times d$, so $d = 6.5$

Down: $\tan\alpha = \frac{5}{12} > \mu$, so it slides back; acceleration $= g(\sin\alpha - \mu\cos\alpha) = \frac{49}{325}$

(b) $v^2 = 2 \times \frac{49}{325} \times 6.5 = 1.96$

$v = \sqrt{1.96}$

Answer: (a) 6.5 m (b) 1.4 m s^{-1}

10 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{2} - 1) = 2 - 4\sqrt{2}$, writing x in the form $p + q\sqrt{2}$.

$x = \dfrac{2 - 4\sqrt{2}}{\sqrt{2} - 1}$

Multiply top and bottom by $\sqrt{2} + 1$.

Numerator: $(2 - 4\sqrt{2})(\sqrt{2} + 1) = -6 - 2\sqrt{2}$

Denominator: $2 - 1 = 1$

Answer: $x = -6 - 2\sqrt{2}$