Day 28 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 7 m s^{-1} and accelerates uniformly. She travels 94.4 m in the next 8 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $94.4 = 7\times8 + \frac{1}{2}\times a\times8^2$, so $a = 1.2$
(b) $v = u + at = 7 + 1.2\times8 = 16.6$
(c) For the midpoint, $s = 47.2$: $v^2 = u^2 + 2as = 7^2 + 2\times1.2\times47.2$
$v^2 = 162.28$

Answer: (a) 1.2 m s^{-2} (b) 16.6 m s^{-1} (c) 12.7 m s^{-1}

2 Factor theorem: roots with surds (All boards)

$f(x) = 2x^3 + 8x^2 - 16$.
(a) Show that $(x + 2)$ is a factor of $f(x)$.
(b) Solve $f(x) = 0$, giving surds in simplest form.

$f(-2) = -16 + 32 - 16 = 0$, so $(x + 2)$ is a factor.
$f(x) = 2(x + 2)(x^2 + 2x - 4)$
$x^2 + 2x - 4 = 0$ gives $x = \frac{-2 \pm \sqrt{20}}{2}$
$\sqrt{20} = 2\sqrt{5}$

Answer: $x = -2$ or $x = -1 \pm \sqrt{5}$

3 Conditional probability and independence (All boards)

$P(B) = 0.1$, $P(A) = 0.8$ and $P(A \mid B) = 0.3$.
(a) Find $P(A \cap B)$ and $P(A \cup B)$.
(b) Find $P(B \mid A)$.
(c) Determine whether A and B are independent.

(a) $P(A \cap B) = P(A \mid B)P(B) = 0.3 \times 0.1 = 0.03$
$P(A \cup B) = 0.8 + 0.1 - 0.03 = 0.87$
(b) $P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{0.03}{0.8} = \frac{3}{80}$
(c) $P(A)P(B) = 0.8 \times 0.1 = 0.08$, which does not equal $P(A \cap B) = 0.03$
So A and B are not independent

Answer: (a) 0.03, 0.87 (b) $\frac{3}{80}$ (c) not independent

4 Final seconds of a fall (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is dropped from rest from the top of a cliff. In the last second before it hits the sea it falls 49 m. Find the height of the cliff. Take $g = 9.8$.

Let the total time of fall be t s. Then $s = 4.9t^2$ for the whole fall.
In the first $(t - 1)$ s the stone falls $4.9(t - 1)^2$.
$4.9t^2 - 4.9(t - 1)^2 = 49$
$4.9(2t - 1) = 49$, so $t = 5$
Height $= 4.9\times5^2 = 148.225$

Answer: 148.225 m

5 Foot of a perpendicular (All boards)

The line l has equation $3x - y = -4$.
(a) Find the coordinates of the foot of the perpendicular from P(3, −3) to l.
(b) Hence find the shortest distance from P to l, in exact form.

Gradient of l = 3, so perpendicular gradient = $-\frac{1}{3}$

Perpendicular through P: $-x - 3y = 6$

Solve with l: $x = -\frac{9}{5}$, $y = -\frac{7}{5}$

Distance^2 = $(3 - (-\frac{9}{5}))^2 + (-3 - (-\frac{7}{5}))^2 = \frac{128}{5}$

Answer: (a) $(-\frac{9}{5}, -\frac{7}{5})$ (b) $\frac{8\sqrt{10}}{5}$

6 Equations with sec x and tan x (All boards)

Solve $2\sec^2\theta + 3\tan\theta - 1 = 0$
for $-180° \le \theta \le 180°$. Give answers to 1 d.p.

Use $\sec^2\theta \equiv 1 + \tan^2\theta$: $2\tan^2\theta + 3\tan\theta + 1 = 0$

$\tan\theta = -\frac{1}{2}$ or $\tan\theta = -1$

$\tan\theta = -\frac{1}{2}$: $\theta = -26.6°, 153.4°$

$\tan\theta = -1$: $\theta = -45.0°, 135.0°$

Answer: $\theta = -45.0°, -26.6°, 135.0°, 153.4°$

7 Bayes' theorem in context (All boards)

3% of people have a disease. The screening test is positive for 90% of those who have it and for 2% of those who do not.
(a) Find the probability that a person tests positive.
(b) Given a positive test, find the probability that the person has the disease (3 s.f.).

(a) P(+) = 0.03 × 0.9 + 0.97 × 0.02
= 0.027 + 0.0194 = 0.0464

(b) P(has it | +) = $\frac{0.03 \times 0.9}{0.0464}$

= 0.582

Answer: (a) 0.0464 (b) 0.582

8 Intersecting or skew lines (CIE (9709); Further Maths on other boards)

l_1: $\mathbf{r} = \begin{pmatrix}0\\7\\4\end{pmatrix} + \lambda\begin{pmatrix}2\\3\\1\end{pmatrix}$

l_2: $\mathbf{r} = \begin{pmatrix}1\\0\\5\end{pmatrix} + \mu\begin{pmatrix}1\\-2\\1\end{pmatrix}$

Determine whether l_1 and l_2 intersect. If they do, find the point of intersection.

Equate components: $0 + 2\lambda = 1 + 1\mu$; $7 + 3\lambda = 0 - 2\mu$; $4 + 1\lambda = 5 + 1\mu$
Solving two of the equations gives a unique (λ, μ)
which does not satisfy the third, so the lines do not meet

Answer: The lines do not intersect (skew)

9 Angle between vectors (3D) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The points A, B and C have coordinates (3, 0, 2), (−1, 0, −1) and (2, 2, 1).
Find angle ABC, in degrees to 1 d.p.

BA = $\begin{pmatrix}4\\0\\3\end{pmatrix}$, BC = $\begin{pmatrix}3\\2\\2\end{pmatrix}$

BA·BC = 18

|BA| = $\sqrt{25}$, |BC| = $\sqrt{17}$

cos ABC = $\frac{18}{\sqrt{25} \times \sqrt{17}}$ = 0.8731, so angle ABC = 29.2°

Answer: 29.2°

10 Partial fractions and binomial expansion (All boards)

f(x) = $\frac{-6x - 7}{(1 + 3x)(1 - 2x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) up to x^2.

(c) State the range of x for which the expansion is valid.

(a) f(x) = $\frac{A}{(1 + 3x)} + \frac{B}{(1 - 2x)}$: A = −3, B = −4

(b) $-3(1 + 3x)^{-1} = -3(1 - 3x + 9x^2 ...)$

Add the series for both fractions.

(c) Need |3x| < 1 and |2x| < 1

Answer: (a) $\frac{-3}{(1 + 3x)} + \frac{-4}{(1 - 2x)}$ **(b)** $-7 + x - 43x^2$ **(c)** $|x| < \frac{1}{3}$