Day 279 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Trigonometric equations (quadratic in sin or cos) (All boards)

Solve $2\cos^2\theta - \sin\theta - 2 = 0$
for $0° \leq \theta \leq 360°$.

Use $\cos^2\theta \equiv 1 - \sin^2\theta$: $2\sin^2\theta + \sin\theta = 0$

Solving the quadratic: $\sin\theta = -\frac{1}{2}$ or $\sin\theta = 0$

$\sin\theta = -\frac{1}{2}$: $\theta = 210°, 330°$

$\sin\theta = 0$: $\theta = 0°, 180°, 360°$

Answer: $\theta = 0°, 180°, 210°, 330°, 360°$

2 Polynomial division (All boards)

Find the quotient and the remainder when $2x^3 + 11x^2 + 11x - 15$ is divided by $(x + 3)$.

Quotient (matching leading terms in turn): $2x^2 + 5x - 4$

$(x + 3)(2x^2 + 5x - 4) = 2x^3 + 11x^2 + 11x - 12$

Remainder $= 2x^3 + 11x^2 + 11x - 15 - (2x^3 + 11x^2 + 11x - 12) = -3$

Answer: quotient $2x^2 + 5x - 4$, remainder -3

3 Newton's second law with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

A particle of mass 2 kg is acted on by forces $F_1 = (7\mathbf{i} + 4\mathbf{j})$ N and F_2. Its acceleration is $(2\mathbf{i} + 4\mathbf{j})$ m s^{-2}. Find (a) F_2, (b) the angle between F_2 and $\mathbf{i}$, to 1 d.p.

$F = ma$: $F_1 + F_2 = 2(2\mathbf{i} + 4\mathbf{j}) = 4\mathbf{i} + 8\mathbf{j}$

(a) $F_2 = (4\mathbf{i} + 8\mathbf{j}) - (7\mathbf{i} + 4\mathbf{j}) = -3\mathbf{i} + 4\mathbf{j}$

(b) $|F_2| = \sqrt{(-3^2 + 4^2)} = 5$

$\cos\theta = -3 \div 5$, so $\theta = 126.9°$

Answer: (a) $F_2 = (-3\mathbf{i} + 4\mathbf{j})$ N (b) 126.9°

4 Sigma notation (All boards)

Given that $\sum_{r=1}^{n}(4r - 2) = 722$, find the value of n.

$$\sum_{r=1}^{n}(4r - 2) = 4 \times \frac{n(n + 1)}{2} - 2n$$

$2n^2 = 722$

$4n^2 - 1444 = 0$

Solve and reject the negative root: $n = 19$

Answer: 19

5 Connected rates of change: cones and bowls (AQA, Edexcel, OCR, MEI, Eduqas)

A hemispherical bowl of radius 6 cm is filled at 50 cm^{3}s^{-1}. When the depth is h, $V = \pi(6h^2 - \frac{1}{3}h^3)$.

Find $\frac{dh}{dt}$ when $h = 2$ cm.

$$\frac{dV}{dh} = \pi(12h - h^2) = \pi h(12 - h)$$

$$\frac{dh}{dt} = \frac{50}{\pi h(12 - h)}$$

$$h = 2: \frac{dh}{dt} = \frac{50}{\pi \times 2 \times 10} = \frac{5}{2\pi} \text{ cm s}^{-1}$$

Answer: $\frac{5}{2\pi}$ cm/s

6 Connected particles (smooth table) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (7 kg) rests on a smooth table, joined by a light string over a smooth pulley at the edge to particle B (1 kg), hanging 1.2 m above the floor. Released from rest. Find (a) the tension, (b) the speed of B as it hits the floor. Take g = 9.8. Give answers to 3 s.f. if not exact.

B: $1g - T = 1a$
A: $T = 7a$
Adding: $1g = 8a$, so $a = 1.225$
(a) $T = 7 \times 1.225 = 8.575$
(b) $v^2 = 0 + 2 \times 1.225 \times 1.2 = 2.94$
Answer: (a) 8.575 N (b) 1.71 m s^{-1}

7 Foot of perpendicular and distance to a line (CIE (9709); Further Maths on other boards)

The line l has equation $\mathbf{r} = \begin{pmatrix} -3 \\ 6 \\ 7 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$.

Find the coordinates of the foot of the perpendicular from P(2, 3, −1) to l, and the exact distance from P to l.

Foot F = A + t d: $\mathbf{PF} = \begin{pmatrix} t - 5 \\ -2t + 3 \\ -2t + 8 \end{pmatrix}$
$\mathbf{PF} \cdot \mathbf{d} = 0$ gives $9t - 27 = 0$, so $t = 3$
F = (0, 0, 1); $\mathbf{PF} = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}$, so $|\mathbf{PF}| = \sqrt{17}$
Answer: F(0, 0, 1), distance $\sqrt{17}$

8 Continuous random variable: cumulative distribution and median (CIE)

The random variable X has pdf $f(x) = \frac{2}{9}(3 - x)$ for $0 \le x \le 3$, and 0 otherwise.

(a) Find the cumulative distribution function F(x) for $0 \le x \le 3$.

(b) Find $P(X > \frac{3}{2})$. (c) Find the median of X to 3 s.f.

(a) $F(x) = \int_0^x f(t)\,dt = \frac{2x}{3} - \frac{x^2}{9}$
(b) $P(X > \frac{3}{2}) = 1 - F(\frac{3}{2}) = \frac{1}{4}$
(c) $F(m) = \frac{1}{2}$: $\frac{2m}{3} - \frac{m^2}{9} = \frac{1}{2}$, so $m = 3(1 - \frac{1}{\sqrt{2}})$
$m = 0.879$
Answer: (b) $\frac{1}{4}$ (c) 0.879

9 Equations quadratic in disguise (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x - 9\sqrt{x} + 20 = 0$.

Let $u = \sqrt{x}$: $u^2 - 9u + 20 = 0$
$(u - 5)(u - 4) = 0$ so $u = 5$ or 4
$\sqrt{x} = 5$ gives $x = 25$; $\sqrt{x} = 4$ gives $x = 16$
Answer: x = 16 or x = 25

10 Tree diagram with conditional probability (All boards)

The tree diagram shows probabilities for events R and S.

(a) Find P(S).

(b) Find P(R | S).

(a) $P(S) = 0.7 \times 0.8 + 0.3 \times 0.3 = 0.65$

(b) $P(R \mid S) = \frac{P(R \cap S)}{P(S)} = \frac{0.7 \times 0.8}{0.65}$

Answer: (a) 0.65 (b) $\frac{56}{65}$