Day 278 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Exact values from the addition formulae (All boards)

By writing 15° as 45° − 30°, show that
tan 15° = 2 − $\sqrt{3}$.

tan 15° = tan(45° − 30°) = $\frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$

tan 30° = $\frac{\sqrt{3}}{3}$, tan 45° = 1, tan 60° = $\sqrt{3}$, tan 120° = $-\sqrt{3}$, tan 150° = $-\frac{\sqrt{3}}{3}$

= $\frac{1 - \frac{\sqrt{3}}{3}}{1 + 1 \times \frac{\sqrt{3}}{3}}$, which rationalises to 2 − $\sqrt{3}$

Answer: $2 - \sqrt{3}$

2 Modelling with a normal distribution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

At one location, the daily mean wind speed is modelled by N(8.2, 2.9^2).
(a) Find the probability that on a randomly chosen day the value exceeds 12 knots.
(b) Estimate the number of such days in a period of 31 days.

(a) $z = \frac{12 - 8.2}{2.9} = 1.31$
P(Z > 1.31) = 0.0950
(b) 31 × 0.0950 = 2.95 ≈ 3
Answer: (a) 0.0950 (b) 3 days

3 Binomial expansion: a coefficient (All boards)

Find the coefficient of x^2 in the expansion of $(-2 - x)^8$.

Term in x^2: ${}^8C_2 \times (-2)^6 \times (-1x)^2$
= 28 × 64 × 1
Answer: 1792

4 Beam on two supports (AQA, Edexcel, OCR, MEI)

A uniform beam AB rests on supports at C and D, with a particle attached at A. Find the reactions at C and D. Take g = 9.8. Give answers to 3 s.f. if not exact.

Moments about C: $R_D \times 3 = 15\,g \times 1.5 - 8\,g \times 1$
$R_D = 47.4$
Resolving vertically: $R_C + R_D = 23\,g = 225.4$
$R_C = 225.4 - 47.4$
Answer: R_C = 178 N, R_D = 47.4 N

5 Inverse of a rational function (All boards)

f(x) = $\frac{-4x + 6}{4x + 4}$, x ∈ ℝ, x ≠ −1.

Find $f^{-1}(x)$ and state its domain.

$y = \frac{-4x + 6}{4x + 4}$: y(4x + 4) = −4x + 6
4xy + 4y = −4x + 6
x(4y + 4) = −4y + 6
$x = \frac{-4y + 6}{4y + 4}$, then swap x and y

Answer: $f^{-1}(x) = \frac{-4x + 6}{4x + 4}$, x ≠ −1

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6 Hypothesis test for a normal mean (All boards)

The mean fill volume of bottles (ml) is normally distributed with standard deviation 2; it was previously 44.
A random sample of 16 has mean 43.5.
Test, at the 1% level, whether the mean has decreased.

$H_0: \mu = 44$, $H_1: \mu < 44$

$\bar{X} \sim N(44, \frac{2^2}{16})$, $z = \frac{43.5 - 44}{\frac{2}{4}} = -1.00$

p-value = 0.159 > 0.01

Do not reject H_0: no evidence that the mean has decreased.

Answer: $z = -1.00$, $p = 0.159$; do not reject H_0

7 Integration by substitution (All boards)

Use the substitution $u = x^2 + 1$ to find the exact value of

$\int_0^2 x(x^2 + 1)^2 \, dx$.

$du = 2x \, dx$, so $x \, dx = \frac{1}{2}du$; $x = 0$ gives $u = 1$, $x = 2$ gives $u = 5$

$I = \frac{1}{2}\int_1^5 u^2 \, du = \frac{1}{2}[\frac{u^3}{3}]$

$= \frac{62}{3}$

Answer: $\frac{62}{3}$

8 Circle and line: chord length (AQA, Edexcel, OCR, MEI, Eduqas)

The line $3x + 4y = 27$ meets the circle $(x + 1)^2 + (y - 3)^2 = 16$ at the points A and B. Find the exact length of AB.

Centre $(-1, 3)$, radius 4. Distance from centre to line:

$d = \frac{|-18|}{5} = \frac{18}{5}$

Half-chord$^2 = 16 - (\frac{18}{5})^2 = \frac{76}{25}$

$AB = 2\sqrt{\frac{76}{25}}$

Answer: $\frac{20\sqrt{19}}{25}$

9 Laws of logarithms in terms of p and q (All boards)

Given that $\log_a 2 = p$ and $\log_a 3 = q$,

express $\log_a(\frac{\sqrt{6}}{8})$ in terms of p and q.

$\frac{\sqrt{6}}{8} = 2^{-\frac{5}{2}} \times 3^{\frac{1}{2}}$

$\log_a(2^{-\frac{5}{2}} \times 3^{\frac{1}{2}}) = -\frac{5}{2}\log_a 2 + \frac{1}{2}\log_a 3$

$= -\frac{5}{2}p + \frac{1}{2}q$

Answer: $-\frac{5}{2}p + \frac{1}{2}q$

10 Closest approach of two particles (Edexcel, OCR, MEI)

At t = 0, P and Q have position vectors $(12\mathbf{i} + 4\mathbf{j})$ m and $(-2\mathbf{i} - \mathbf{j})$ m and velocities $(2\mathbf{i} - 2\mathbf{j})$ m s^{-1} and $(3\mathbf{i} + 2\mathbf{j})$ m s^{-1}. Find (a) the time when they are closest, (b) the least distance between them. Give answers to 3 s.f. if not exact.

$\mathrm{PQ} = \mathrm{r_P} - \mathrm{r_Q} = (14\mathbf{i} + 5\mathbf{j}) + \mathrm{t}(-\mathbf{i} - 4\mathbf{j})$

$\mathrm{d}^2 = (-\mathrm{t} + 14)^2 + (-4\mathrm{t} + 5)^2 = 17\mathrm{t}^2 - 68\mathrm{t} + 221$

(a) $\mathrm{d}(\mathrm{d}^2)/\mathrm{dt} = 0$: $34\ \mathrm{t} + -68 = 0$, so $\mathrm{t} = 2$

(b) At t = 2: $\mathrm{PQ} = 12\mathbf{i} - 3\mathbf{j}$, so $\mathrm{d} = \sqrt{153}$

Answer: (a) t = 2 s (b) 12.4 m