Day 277 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Discrete random variable with unknown constant (Edexcel, OCR, MEI, Eduqas, CIE)

The random variable X has $P(X = x) = k(x^2 + 1)$ for $x = 0, 1, 2, 3, 4$.

(a) Show that $k = \frac{1}{35}$. (b) Find E(X) and Var(X).

(c) Find $P(X \geq 1)$.

(a) $\Sigma P(X = x) = 1$: $1 + 2 + 5 + 10 + 17 = 35$, so $35k = 1$

(b) $E(X) = 1 \times \frac{2}{35} + 2 \times \frac{1}{7} + 3 \times \frac{2}{7} + 4 \times \frac{17}{35} = \frac{22}{7}$

$E(X^2) = \frac{384}{35}$, so $Var(X) = \frac{384}{35} - (\frac{22}{7})^2 = \frac{268}{245}$

(c) $P(X \geq 1) = \frac{2}{35} + \frac{1}{7} + \frac{2}{7} + \frac{17}{35} = \frac{34}{35}$

Answer: (b) E(X) = $\frac{22}{7}$, Var(X) = $\frac{268}{245}$ (c) $\frac{34}{35}$

2 Equations with sin 2x and a trig function (All boards)

Solve $\sin 2x = \sqrt{3} \cos x$ for $0 \leq x \leq 2\pi$.

$\sin 2x \equiv 2\sin x \cos x$: $\cos x (2\sin x - \sqrt{3}) = 0$

$\cos x = 0$: $x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\sin x = \frac{\sqrt{3}}{2}$: $x = \frac{\pi}{3}, \frac{2\pi}{3}$

Do not divide by cos x, as that would lose solutions

Answer: x = $\frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{2}$

3 Projectile from a cliff top (AQA, Edexcel, OCR, MEI, Eduqas)

A stone is projected from a cliff top 140.4 m above the sea at 10 m s^{-1}, at angle α above the horizontal, $\tan \alpha = \frac{3}{4}$. Find (a) the time to reach the sea, (b) the horizontal distance travelled, (c) the speed on impact. Take g = 9.8. Give answers to 3 s.f. if not exact.

Horizontal component 8, vertical component 6 (upwards)

(a) $-140.4 = 6t - 4.9t^2$, so $4.9t^2 - 6t - 140.4 = 0$

$t = [6 + \sqrt{2787.84}] \div 9.8 = 6$ (taking the positive root)

(b) Distance = 8×6

(c) Vertical speed = $6 - 9.8 \times 6 = -52.8$; speed = $\sqrt{(8^2 + 52.8^2)}$

Answer: (a) 6 s (b) 48 m (c) 53.4 m s^{-1}

4 Line and curve intersection (diagram) (All boards)

The diagram shows the curve $y = x^2 - x - 2$ and the line $y = 2x + 2$, meeting at A and B.

(a) Find the coordinates of A and B.

(b) Find the coordinates of the midpoint of AB.

$x^2 - x - 2 = 2x + 2$: $x^2 - 3x - 4 = 0$

$(x + 1)(x - 4) = 0$, $x = -1$ or 4

A(−1, 0), B(4, 10)

Midpoint: average the coordinates

Answer: (a) A(−1, 0), B(4, 10) (b) ($\frac{3}{2}$, 5)

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5 Work-energy on a rough slope (CIE)

A 8 kg block is released from rest on a rough plane inclined at α, $\tan\alpha = \frac{4}{3}$, $\mu = 0.5$, and slides 10 m down a line of greatest slope. Use work and energy to find (a) the work done against friction, (b) its speed after 10 m. Take $g = 10$.

$R = 8g\cos\alpha = 48$ N, $F = \mu R = 24$ N
(a) Work done against friction $= F \times d = 24 \times 10$
(b) Loss of PE $= 8g \times 10\sin\alpha = 640$ J
Work-energy: $\frac{1}{2}\times 8 \times v^2 = 640 - 240 = 400$
$v^2 = 100$

Answer: (a) 240 J (b) 10 m s^{-1}

6 Conditional probability without replacement (All boards)

A bag has 5 green and 2 yellow counters. Two are taken at random without replacement.
Given that the second counter is green, find the probability that the first was green.

$P(\text{green then green}) = \frac{5}{7} \times \frac{4}{6} = \frac{10}{21}$

$P(\text{yellow then green}) = \frac{2}{7} \times \frac{5}{6} = \frac{5}{21}$

$P(\text{second green}) = \frac{10}{21} + \frac{5}{21} = \frac{5}{7}$

$P(\text{first green} \mid \text{second green}) = \frac{\frac{10}{21}}{\frac{5}{7}}$

$= \frac{2}{3}$

Answer: $\frac{2}{3}$

7 Equations with cosec x and cot x (All boards)

Solve $3\text{cosec}^2\theta + 6\cot\theta - 27 = 0$
for $-180° < \theta < 180°$, $\theta \neq 0$. Give answers to 1 d.p.

Use $\text{cosec}^2\theta \equiv 1 + \cot^2\theta$: $3\cot^2\theta + 6\cot\theta - 24 = 0$
$\cot\theta = 2$ or $\cot\theta = -4$

$\cot\theta = 2$ means $\tan\theta = \frac{1}{2}$: $\theta = -153.4°, 26.6°$

$\cot\theta = -4$ means $\tan\theta = -\frac{1}{4}$: $\theta = -14.0°, 166.0°$

Answer: $\theta = -153.4°, -14.0°, 26.6°, 166.0°$

8 Periodic sequences (All boards)

A sequence is defined by $u_1 = -3$ and $u_{n+1} = \frac{1}{1 - u_n}$.

(a) Find u_2 and u_3.

(b) Find u_{50} and $\sum_{r=1}^{50} u_r$.

$u_2 = \frac{1}{4}$, $u_3 = \frac{4}{3}$, $u_4 = -3$

$u_4 = u_1$, so the sequence repeats with period 3.

$50 = 3 \times 16 + 2$, so $u_{50} = u_2 = \frac{1}{4}$

Sum of one period $= -\frac{17}{12}$; 16 full periods plus 2 extra term(s)

Answer: (a) $\frac{1}{4}$, $\frac{4}{3}$ (b) $u_{50} = \frac{1}{4}$, sum $= -\frac{305}{12}$

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9 Binomial expansion: terms with negative powers of x (All boards)

Find the coefficient of x^6 in the expansion of

$(2x^2 - \frac{2}{x^2})^7$.

General term: ${}^7C_r\,(2x^2)^{7-r}\,(-2x^{-2})^r$

Power of x: $2(7 - r) - 2r = 6$

$14 - 4r = 6$ so $r = 2$

Coefficient = ${}^7C_2 \times 2^5 \times (-2)^2 = 2688$

Answer: 2688

10 Trapezium rule with over or under estimate (AQA, Edexcel, OCR, MEI, Eduqas)

Use the trapezium rule with 4 strips to estimate $\int_0^4 3(\ln(x + 2))\,dx$, giving your answer to 3 s.f.

State, with a reason, whether your estimate is an overestimate or an underestimate.

h = 1; y-values at x = 0, 1, 2, 3, 4: 2.0794, 3.2958, 4.1589, 4.8283, 5.3753

$T = \frac{1}{2}[2.0794 + 5.3753 + 2(3.2958 + 4.1589 + 4.8283)]$

T = 16.0

The curve is concave (y'' < 0) on this interval, so the chords lie below the curve: underestimate

Answer: 16.0, an underestimate