Day 276 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Arithmetic sequences: finding a and d (All boards)

The 5th term of an arithmetic sequence is -2 and the 8th term is -14.
Find the first term and common difference, and the sum of the first 20 terms.

$u_8 - u_5 = 3\,d = -14 - (-2)$ so $d = -4$

$a + 4\,d = -2$ so $a = 14$

$S_{20} = \frac{20}{2}(2 \times 14 + 19 \times (-4))$

Answer: $a = 14$, $d = -4$, $S_{20} = -480$

2 Connected rates of change: cones and bowls (AQA, Edexcel, OCR, MEI, Eduqas)

Water pours into a cone, vertex down, at 30 cm^3s^{-1}. The surface radius is always $\frac{h}{3}$.

Find the rate at which the depth is rising when $h = 4$ cm.

$V = \frac{1}{3}\pi(h/3)^2h = \frac{\pi h^3}{27}$, so $\frac{dV}{dh} = \frac{\pi h^2}{9}$

$\frac{dh}{dt} = \frac{dV}{dt \div dV/dh} = \frac{30 \times 9}{\pi h^2}$

$h = 4$: $\frac{dh}{dt} = \frac{135}{8\pi}$ cm s^{-1}

Answer: $\frac{135}{8\pi}$ cm/s

3 Circle: centre, radius and position of a point (AQA, Edexcel, OCR, MEI, Eduqas)

A circle has equation $x^2 + y^2 + 6x + 4y - 12 = 0$.
(a) Find the centre and radius of the circle.
(b) Determine whether P$(-5, -3)$ lies inside or outside the circle.

Complete the square: $(x + 3)^2 + (y + 2)^2 = 25$

Centre $(-3, -2)$, radius 5

Distance^2 from centre to P $= (-2)^2 + (-1)^2 = 5$

Compare with $r^2 = 25$

Answer: (a) centre $(-3, -2)$, radius 5 (b) P is inside the circle

4 Particles on two smooth inclined planes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The system is released from rest. Find (a) the magnitude of the acceleration and which particle moves down, (b) the tension. Take $g = 9.8$. Give answers to 3 s.f. if not exact.

A: weight component along plane $= 5\,g \sin 30°$; B: $3\,g \sin 30°$

A moves down, B up ($5 \sin 30° > 3 \sin 30°$): $F = ma$ for the system

$5\,g \sin 30° - 3\,g \sin 30° = (5 + 3)a$

(a) $a = 1.22$

(b) For A: $5\,g \sin 30° - T = 5\,a$, so $T = 18.4$

Answer: (a) 1.225 m s^{-2}, A moves down (b) 18.4 N

5 Normal approximation to the binomial (All boards)

$X \sim B(120, 0.2)$. Use a normal approximation to estimate $P(X \leq 26)$ to 3 significant figures.

$np = 24$, $npq = 19.2$, so $Y \sim N(24, 19.2)$

With continuity correction: $P(Y < 26.5)$

$z = \frac{26.5 - 24}{4.38} = 0.571$, $P(Z < 0.571) = 0.716$

Answer: 0.716

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6 Integration by substitution (All boards)

Use the substitution u = cos x to find the exact value of

$\int_0^{\frac{\pi}{2}} \sin x \cos^4 x \, dx$.

du = $-\sin x \, dx$; x = 0 gives u = 1, x = $\frac{\pi}{2}$ gives u = 0

$I = \int_1^0 -u^4 \, du = \int_0^1 u^4 \, du = [\frac{u^5}{5}]_0^1$

$= \frac{1}{5}$

Answer: $\frac{1}{5}$

7 Poisson over different intervals (CIE)

Faults are reported at random at an average of 10 per hour.
Find the probability that in a 30-minute period there are (a) zero events (b) at least 1.
(c) Find the probability that at least one occurs in each of two separate 30-minute periods.

$\lambda = 10 \times \frac{30}{60} = 5$, X ~ Po(5)

(a) $P(X = 0) = e^{-5} = 0.00674$
(b) $P(X \geq 1) = 1 - P(X \leq 0) = 0.993$
(c) $(1 - 0.00674)^2 = 0.987$
Answer: (a) 0.00674 (b) 0.993 (c) 0.987

8 Partial fractions: repeated factor (All boards)

Express $\frac{-2x^2 + 17x - 17}{(x + 1)(x - 2)^2}$ in the form $\frac{A}{(x + 1)} + \frac{B}{(x - 2)} + \frac{C}{(x - 2)^2}$, and find A, B and C.

$-2x^2 + 17x - 17 \equiv A(x - 2)^2 + B(x + 1)(x - 2) + C(x + 1)$
x = −1: $-36 = A(-3)^2$ so A = −4
x = 2: 9 = C(3) so C = 3
Coefficient of x^2: −2 = A + B so B = 2
Answer: A = −4, B = 2, C = 3

9 Particle supported by two strings (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 5 kg particle hangs in equilibrium from two light strings, as shown. Find the tension in each string (T_1 in the string at angle α). Take g = 9.8.

sin α = 0.8, cos α = 0.6, sin β = 0.6, cos β = 0.8
Horizontal: $T_1 \cos \alpha = T_2 \cos \beta$, so $T_2 = 0.75 \, T_1$
Vertical: $T_1 \sin \alpha + T_2 \sin \beta = 5 \, g = 49$
$T_1(0.8 + 0.45) = 49$
$T_1 = 39.2$, $T_2 = 29.4$
Answer: T_1 = 39.2 N, T_2 = 29.4 N

10 Parametric differentiation: tangent and normal (All boards)

A curve has parametric equations $x = t^2 + 1$, $y = t^3 - t$.

Find $\frac{dy}{dx}$ in terms of t, and the equation of the normal at the point where t = −2, in the form ax + by + c = 0 (integers).

$\frac{dy}{dx} = \frac{dy}{dt \div dx/dt} = \frac{3t^2 - 1}{2t}$

t = −2: point (5, −6), gradient $-\frac{11}{4}$

Normal gradient = $-1 \div (-\frac{11}{4}) = \frac{4}{11}$, so normal: 4x − 11y − 86 = 0

Answer: $\frac{dy}{dx} = \frac{3t^2 - 1}{2t}$**; 4x − 11y − 86 = 0**