Day 275 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Quotient rule: stationary points (All boards)

Find the coordinates of the stationary points of

$y = \frac{x^2 + 7}{x + 3}$.

$\frac{dy}{dx} = \frac{(x + 3) \times 2x - (x^2 + 7) \times 1}{(x + 3)^2} = \frac{x^2 + 6x - 7}{(x + 3)^2}$

$\frac{dy}{dx} = 0$: $x^2 + 6x - 7 = 0$, so $(x - 1)(x + 7) = 0$

$x = 1$: $y = 2$; $x = -7$: $y = -14$

Answer: $(-7, -14)$, $(1, 2)$

2 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 6 m s^{-1} and accelerates uniformly. She travels 70 m in the next 10 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $70 = 6 \times 10 + \frac{1}{2} \times a \times 10^2$, so $a = 0.2$

(b) $v = u + at = 6 + 0.2 \times 10 = 8$

(c) For the midpoint, $s = 35$: $v^2 = u^2 + 2as = 6^2 + 2 \times 0.2 \times 35$

$v^2 = 50$

Answer: (a) 0.2 m s^{-2} (b) 8 m s^{-1} (c) 7.07 m s^{-1}

3 Index equations solved with logarithms (All boards)

Solve $3^{x-1} = 7^{x-3}$.

Give your answer to 3 significant figures.

Take logs: $(x - 1) \ln 3 = (x - 3) \ln 7$

Collect x terms: $x(\ln 3 - \ln 7) = -3 \ln 7 + \ln 3$

$x = \frac{-3 \ln 7 + \ln 3}{\ln 3 - \ln 7}$

$x = 5.59$

Answer: x = 5.59

4 Normal distribution probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the mass (g), is normally distributed with mean 62 and standard deviation 5.

Find (a) P(X > 59.5) (b) P(59.5 < X < 68.5), each to 3 s.f.

(a) $z = \frac{59.5 - 62}{5} = -0.500$, $P(Z > -0.500) = 0.691$

(b) $z_1 = -0.500$, $z_2 = \frac{68.5 - 62}{5} = 1.30$

$P(59.5 < X < 68.5) = \Phi(1.30) - \Phi(-0.500) = 0.595$

Answer: (a) 0.691 (b) 0.595

5 Parametric equations: points on axes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 3t^2 - 5$, $y = -t(t^2 - 4)$.

Find the coordinates of the points where the curve meets the x-axis.

$y = 0$: $t = 0$ or $t^2 = 4$, so $t = 0, 2, -2$

$t = 0$: $x = -5$

$t = \pm 2$: $x = 7$

Answer: $(-5, 0)$ and $(7, 0)$

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6 Partial fractions and integration (All boards)

(a) Express $\frac{-x-1}{(x+3)^2}$ in the form $\frac{A}{x+3}+\frac{B}{(x+3)^2}$.

(b) Hence find the exact value of $\int_0^2 \frac{-x-1}{(x+3)^2}\,dx$.

$-x-1 \equiv A(x+3)+B$, so $A=-1$ and $B=2$

$I = [-1\ln(x+3) - \frac{2}{x+3}]_0^2$

$= -1\ln\frac{5}{3} + 2(\frac{1}{3}-\frac{1}{5})$

$= \frac{15\ln 3 + 4 - 15\ln 5}{15}$

Answer: (a) $A=-1$, $B=2$; (b) $\frac{15\ln 3 + 4 - 15\ln 5}{15}$

7 Hypothesis test for a Poisson mean (CIE)

Accidents occur as a Poisson process at a mean rate of 5 per week.
After a change, the number recorded in one week is 9.
Test at the 5% level whether the mean rate has increased.

$H_0: \lambda = 5$, $H_1: \lambda > 5$; $X \sim Po(5)$

$P(X \geq 9) = 1 - P(X \leq 8) = 0.0681$

$0.0681 > 0.05$

Do not reject H_0: insufficient evidence of a change.

Answer: p-value 0.0681; do not reject H_0

8 Recurrence relations: limits (All boards)

$u_1 = 12$ and $u_{n+1} = -\frac{1}{2}u_n + 9$.

(a) Find the limit L of the sequence as $n \to \infty$.
(b) Find the least n for which $|u_n - L| < 0.01$.

(a) $L = -\frac{1}{2}L + 9$ gives $L = \frac{9}{1-(-\frac{1}{2})} = 6$

(b) $u_n - L = (u_1 - L)\,r^{n-1} = 6 \times (-\frac{1}{2})^{n-1}$

$|6| \times \frac{1}{2}^{n-1} < \frac{1}{100}$

Check: n = 10 gives 0.01172, n = 11 gives 0.00586

Answer: (a) L = 6 (b) n = 11

9 Closest approach of two particles (Edexcel, OCR, MEI)

At t = 0, P and Q have position vectors $(15\mathbf{i} + 12\mathbf{j})$ m and $(4\mathbf{i} + 5\mathbf{j})$ m and velocities $(-5\mathbf{i} - 4\mathbf{j})$ m s^{-1} and $(-2\mathbf{i} - 3\mathbf{j})$ m s^{-1}. Find (a) the time when they are closest, (b) the least distance between them. Give answers to 3 s.f. if not exact.

$PQ = r_P - r_Q = (11\mathbf{i} + 7\mathbf{j}) + t(-3\mathbf{i} - \mathbf{j})$

$d^2 = (-3t + 11)^2 + (-t + 7)^2 = 10t^2 - 80t + 170$

(a) $d(d^2)/dt = 0$: $20t + -80 = 0$, so $t = 4$

(b) At t = 4: $PQ = -\mathbf{i} + 3\mathbf{j}$, so $d = \sqrt{10}$

Answer: (a) t = 4 s (b) 3.16 m

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10 Geometric series: smallest n (All boards)

A salary is £25000 in year 1 and increases by 2% each year.
In which year does the salary first exceed £62500?

Salary in year n = $25000 \times 1.02^{n-1}$

$1.02^{n-1} > \frac{5}{2}$

$n - 1 > \frac{\log \frac{5}{2}}{\log 1.02} = 46.27$

$n = 48$

Answer: year 48