Day 274 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Indices: solving equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $8^{3x-3} = 16^{2x+3}$.

Write both sides as powers of 2.

$(2^3)^{3x-3} = (2^4)^{2x+3}$

$3(3x - 3) = 4(2x + 3)$

$9x - 9 = 8x + 12$

$x = 21$

Answer: x = 21

2 Exact definite integrals of standard functions (All boards)

Find the exact value of

$\int_1^2 \left(\frac{4}{x} + \frac{1}{x^2}\right) dx$.

Integrate: $[4 \ln x - \frac{1}{x}]$

Substitute the limits and subtract: $\frac{8 \ln 2 + 1}{2}$

Answer: $\frac{8 \ln 2 + 1}{2}$

3 Inverse normal (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the score (marks), is N(54, 12^2). The highest 10% of values exceed k.
Find k to 3 significant figures.

P(X > k) = 0.1, so P(X < k) = 0.9

$z = \Phi^{-1}(0.9) = 1.28$

$k = 54 + 1.28 \times 12 = 69.4$

Answer: 69.4

4 Integration by substitution (All boards)

Use the substitution u = ln x to find the exact value of

$\int_1^e \frac{(\ln x)^2}{x} dx$.

$du = \frac{1}{x}dx$; x = 1 gives u = 0, x = e gives u = 1

$I = \int_0^1 u^2 \, du = [\frac{u^3}{3}]$

$= \frac{1}{3}$

Answer: $\frac{1}{3}$

5 Equilibrium on a slope: range of force (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 3 kg particle rests on a rough plane inclined at α, $\tan \alpha = \frac{12}{5}$, μ = 0.4. A force of P N acts up a line of greatest slope and the particle stays at rest. Find the range of values of P. Take g = 9.8. Give answers to 3 s.f. if not exact.

$\sin \alpha = \frac{12}{13}$, $\cos \alpha = \frac{5}{13}$, $R = 3g \cos \alpha = \frac{147}{13}$ N

Limiting friction = $\mu R = \frac{294}{65}$ N

P smallest: friction acts up the slope: $P + \mu R = 3g \sin \alpha$, $P = \frac{1764}{65} - \frac{294}{65}$

P greatest: friction acts down the slope: $P = \frac{1764}{65} + \frac{294}{65}$

Answer: 22.6 N ≤ P ≤ 31.7 N

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6 Parametric differentiation with trigonometry (All boards)

A curve has parametric equations $x = 3\sec t$, $y = 3\tan t$.

Find $\frac{dy}{dx}$ in terms of t and the exact equation of the tangent at the point where $t = \frac{\pi}{3}$, in the form $y = mx + c$.

$\frac{dy}{dx} = \frac{dy}{dt \div dx/dt} = \frac{3}{3\sin t}$

$t = \frac{\pi}{3}$: gradient $m = \frac{2\sqrt{3}}{3}$, point $(6, 3\sqrt{3})$

$c = y - mx = -\sqrt{3}$

Answer: $\frac{dy}{dx} = \frac{3}{3\sin t}$; $y = \frac{2\sqrt{3}}{3}x - \sqrt{3}$

7 Beam about to tilt (AQA, Edexcel, OCR, MEI)

A uniform plank AB (mass 10 kg, length 4 m) rests horizontally on supports at C and D, where AC = 1 m and DB = 1.5 m. A 5 kg particle is placed at E, AE = x m, and the plank is about to tilt about D. Find x.

About to tilt about D, so the reaction at C is zero and E lies between D and B.
Moments about D: $5g \times (x - 2.5) = 10g \times (2.5 - 2)$
$5(x - 2.5) = 5$
$x = 2.5 + 1$
Answer: x = 3.5

8 Circle touching both axes (AQA, Edexcel, OCR, MEI, Eduqas)

A circle touches both coordinate axes and passes through the point $(-8, -1)$. Its centre lies in the third quadrant. Find the equations of the two possible circles.

Centre $(-r, -r)$, radius $r > 0$.
Through $(-8, -1)$: $(-8 + r)^2 + (-1 + r)^2 = r^2$
$r^2 - 18r + 65 = 0$
$(r - 5)(r - 13) = 0$, so r = 5 or 13
Answer: $(x + 5)^2 + (y + 5)^2 = 25$ **or** $(x + 13)^2 + (y + 13)^2 = 169$

9 Circle and line: chord length (AQA, Edexcel, OCR, MEI, Eduqas)

The line $-4x + 3y = 48$ meets the circle $(x + 2)^2 + (y - 2)^2 = 49$ at the points A and B. Find the exact length of AB.

Centre $(-2, 2)$, radius 7. Distance from centre to line:
$d = \frac{|-34|}{5} = \frac{34}{5}$

Half-chord$^2 = 49 - (\frac{34}{5})^2 = \frac{69}{25}$

$AB = 2\sqrt{\frac{69}{25}}$

Answer: $\frac{10\sqrt{69}}{25}$

10 Continuous random variable: variance (CIE)

X has pdf $f(x) = \frac{2}{16}x$ for $0 \le x \le 4$, and 0 otherwise.

Find E(X), Var(X) and the standard deviation of X (3 s.f.).

$E(X) = \int_0^4 x\, f(x)\, dx = \frac{8}{3}$

$E(X^2) = \int_0^4 x^2\, f(x)\, dx = 8$

$Var(X) = 8 - (\frac{8}{3})^2 = \frac{8}{9}$

Standard deviation $= \sqrt{\frac{8}{9}} = 0.943$

Answer: $E(X) = \frac{8}{3}$, $Var(X) = \frac{8}{9}$, **sd 0.943**