Day 273 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Binomial probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(18, 0.15). Find, to 3 significant figures:
(a) P(X = 5) (b) P(X ≥ 5) (c) P(X ≤ 4)

(a) $C(18, 5)(0.15)^5(0.85)^{13} = 0.0787$
(b) $1 - P(X \leq 4) = 1 - 0.879 = 0.121$
(c) $P(X \leq 4) = 0.879$
Answer: (a) 0.0787 (b) 0.121 (c) 0.879

2 Binomial expansion: a coefficient (All boards)

Find the coefficient of x^4 in the expansion of $(4 + 3x)^8$.

Term in x^4: ${}^8C_4 \times 4^4 \times (3x)^4$
$= 70 \times 256 \times 81$
Answer: 1451520

3 Arc length and sector area (All boards)

A sector of a circle of radius 9 cm has angle 135°.
Find, in terms of π, the arc length, the area and the perimeter of the sector.

$\theta = 135° = \frac{3\pi}{4}$ radians

Arc length $= r\theta = 9 \times \frac{3\pi}{4} = \frac{27\pi}{4}$ cm

Area $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 9^2 \times \frac{3\pi}{4} = \frac{243\pi}{8}$ cm^2

Perimeter $= 2r + \text{arc} = \frac{27\pi + 72}{4}$ cm

Answer: arc $\frac{27\pi}{4}$ cm, area $\frac{243\pi}{8}$ cm^2, perimeter $\frac{27\pi + 72}{4}$ cm

4 Inverse of a rational function (All boards)

$f(x) = \frac{x + 2}{3x - 1}$, $x \in \mathbb{R}$, $x \neq \frac{1}{3}$.

Find $f^{-1}(x)$ and state its domain.

$y = \frac{x + 2}{3x - 1}$: $y(3x - 1) = x + 2$
$3xy - y = x + 2$
$x(3y - 1) = y + 2$
$x = \frac{y + 2}{3y - 1}$, then swap x and y

Answer: $f^{-1}(x) = \frac{x + 2}{3x - 1}$, $x \neq \frac{1}{3}$

5 Newton's second law with a time-varying force (AQA, Edexcel, OCR, MEI)

A 3 kg particle moves on a smooth horizontal line. At time t s a single force of (15 − 3t) N acts along the line. When t = 0 its speed is 2 m s^{-1}. Find (a) its speed when t = 3, (b) its greatest speed.

F = ma: $a = (-3t + 15) \div 3$
$v = \int a\,dt = 5t - 0.5t^2 + c$; v = 2 when t = 0
(a) v(3) = 12.5
(b) Greatest speed when a = 0: t = 5
v(5) = 14.5
Answer: (a) 12.5 m s^{-1} (b) 14.5 m s^{-1}

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6 Regression with coded variables (AQA, Edexcel, OCR, MEI, Eduqas)

A regression line of v on w is $v = 5005 + 25w$, where $w = \frac{t - 2000}{10}$.

(a) Find the regression line of v on t in the form $v = p + qt$.

(b) Estimate v when $t = 2040$.

$v = 5005 + 25 \times \frac{t - 2000}{10}$

$= 5005 + 2.5t - 5000 = 5 + 2.5t$

(b) $v = 5 + 2.5 \times 2040 = 5105$

Answer: (a) $v = 5 + 2.5t$ (b) 5105

7 Implicit differentiation: products and cubes (All boards)

A curve has equation $x^2y - y^3 + 3x = -21$.

Find $\frac{dy}{dx}$ in terms of x and y, and the gradient of the normal at (1, 3).

Differentiate: $2xy + x^2\frac{dy}{dx} - 3y^2\frac{dy}{dx} + 3 = 0$

$\frac{dy}{dx} = -\frac{2xy + 3}{x^2 - 3y^2}$

At (1, 3): $\frac{dy}{dx} = \frac{9}{26}$, so the normal has gradient $-\frac{26}{9}$

Answer: $\frac{dy}{dx} = -\frac{2xy + 3}{x^2 - 3y^2}$; normal gradient $-\frac{26}{9}$

8 Work-energy on a rough slope (CIE)

A 6 kg block is released from rest on a rough plane inclined at α, $\tan\alpha = \frac{3}{4}$, $\mu = 0.3$, and slides 5 m down a line of greatest slope. Use work and energy to find (a) the work done against friction, (b) its speed after 5 m. Take $g = 10$.

$R = 6g\cos\alpha = 48$ N, $F = \mu R = 14.4$ N

(a) Work done against friction $= F \times d = 14.4 \times 5$

(b) Loss of PE $= 6g \times 5\sin\alpha = 180$ J

Work-energy: $\frac{1}{2} \times 6 \times v^2 = 180 - 72 = 108$

$v^2 = 36$

Answer: (a) 72 J (b) 6 m s^{-1}

9 Periodic sequences (All boards)

A sequence is defined by $u_1 = \frac{5}{2}$ and $u_{n+1} = \frac{u_n + 1}{u_n - 1}$.

(a) Find u_2 and u_3.

(b) Find u_{50} and $\sum_{r=1}^{50} u_r$.

$u_2 = \frac{7}{3}$, $u_3 = \frac{5}{2}$, $u_4 = \frac{7}{3}$

$u_3 = u_1$, so the sequence repeats with period 2.

$50 = 2 \times 25 + 0$, so $u_{50} = u_2 = \frac{7}{3}$

Sum of one period $= \frac{29}{6}$; 25 full periods plus 0 extra term(s)

Answer: (a) $\frac{7}{3}$, $\frac{5}{2}$ (b) $u_{50} = \frac{7}{3}$, sum $= \frac{725}{6}$

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10 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $3x^2 + 2xy + 3y^2 = 96$.
Find the coordinates of the points where the tangent is parallel to the x-axis.

$$\frac{dy}{dx} = -\frac{6x + 2y}{2x + 6y}$$

Horizontal tangent: $6x + 2y = 0$, so $y = -3x$

Substitute: $24x^2 = 96$, so $x^2 = 4$ and $x = \pm 2$

Points: $(2, -6)$ and $(-2, 6)$

Answer: $(2, -6)$ and $(-2, 6)$