Day 272 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Venn diagrams and conditional probability (All boards)

P(A) = 0.5, P(B) = 0.55 and P(A ∩ B) = 0.2.
Find
(a) P(A ∪ B)
(b) P(A | B)
(c) P(B | A)

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.55 - 0.2 = 0.85$

(b) $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.55} = \frac{4}{11}$

(c) $P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{0.2}{0.5} = 0.4$

Answer: (a) 0.85 (b) $\frac{4}{11}$ (c) 0.4

2 Binomial expansion: a coefficient (All boards)

Find the coefficient of x^3 in the expansion of $(4 - x)^8$.

Term in x^3: ${}^8C_3 \times 4^5 \times (-1x)^3$
$= 56 \times 1024 \times -1$
Answer: −57344

3 Hinged rod held by a string (AQA, Edexcel, OCR, MEI)

A uniform rod AB (6 kg, 4 m) is smoothly hinged at A to a vertical wall, with a 6 kg particle at B. A light string from B to the wall above A holds the rod horizontal, at angle θ to it, $\tan\theta = \frac{3}{4}$. Find (a) the tension, (b) the magnitude of the hinge force. Take g = 9.8. Give answers to 3 s.f. if not exact.

Moments about A: $T \sin\theta \times 4 = 6g \times 2 + 6g \times 4$
$T \sin\theta = (3 + 6)g = 88.2$, so (a) $T = 147$
Horizontal component of hinge force: $X = T\cos\theta = 118$
Vertical: $Y + T\sin\theta = (6 + 6)g$, so $Y = 6g \div 2 = 29.4$
(b) Force $= \sqrt{(X^2 + Y^2)} = \sqrt{(118^2 + 29.4^2)}$
Answer: (a) 147 N (b) 121 N

4 Newton-Raphson method (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^2 - 5$. Show that the Newton-Raphson formula gives $x_{n+1} = \frac{1}{2}(x_n + \frac{5}{x_n})$,

and use it with $x_0 = \frac{5}{2}$ to find x_2 as an approximation to $\sqrt{5}$, to 4 d.p.

$f'(x) = 2x$, so $x_{n+1} = x_n - \frac{x_n^2 - 5}{2x_n} = \frac{1}{2}(x_n + \frac{5}{x_n})$

$x_1 = \frac{1}{2}(\frac{5}{2} + \frac{5}{\frac{5}{2}}) = \frac{9}{4}$

$x_2 = 2.2361$
Answer: $x_2 = 2.2361$

5 Recurrence relations: unknown first term (All boards)

A sequence is defined by $u_1 = k$ and $u_{n+1} = -2u_n + 5$.

Given that $u_3 = -21$, find (a) the value of k, (b) $\sum_{r=1}^{4} u_r$.

$u_2 = -2k + 5$, $u_3 = (-2)(-2k + 5) + 5$
$4k - 5 = -21$ so $k = -4$
Terms: −4, 13, −21, 47
Sum = 35
Answer: (a) k = −4 (b) 35

Day 272 **A Level Maths: A Tier 10-a-day: Model Solutions**

6 Implicit differentiation: products and cubes (All boards)

A curve has equation $2x^2y - 2y^3 + 4x = -16$.

Find $\frac{dy}{dx}$ in terms of x and y, and the gradient of the normal at (0, 2).

Differentiate: $4xy + 2x^2\frac{dy}{dx} - 6y^2\frac{dy}{dx} + 4 = 0$

$\frac{dy}{dx} = -\frac{4xy + 4}{2x^2 - 6y^2}$

At (0, 2): $\frac{dy}{dx} = \frac{1}{6}$, so the normal has gradient -6

Answer: $\mathbf{\frac{dy}{dx} = -\frac{4xy + 4}{2x^2 - 6y^2}}$**; normal gradient** $\mathbf{-6}$

7 Inverse of a quadratic function (All boards)

$f(x) = 2x^2 + 16x + 35$, $x \geq -4$.

Find $f^{-1}(x)$ and state its domain.

$f(x) = 2(x + 4)^2 + 3$

Let $y = f(x)$: $(x + 4)^2 = \frac{y - 3}{2}$

$x \geq -4$, so take the positive root: $x = -4 + \sqrt{\frac{y - 3}{2}}$

Answer: $\mathbf{f^{-1}(x) = -4 + \sqrt{\frac{x - 3}{2}}}$**,** $\mathbf{x \geq 3}$

8 Catching up (delayed start) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes P at constant speed 20 m s^{-1}. 5 s later car B leaves P from rest, in the same direction, with acceleration 2.5 m s^{-2}. B overtakes A at Q. Find (a) the time B takes to reach Q, (b) PQ, (c) the speed of B at Q.

Let B take s seconds; A has then been moving for (s + 5) seconds.
B: $\frac{1}{2}\times 2.5\times s^2 = 1.25\ s^2$ A: $20(s + 5)$
$1.25\ s^2 - 20\ s - 100 = 0$ so $s = 20$ ($s > 0$)
(b) $PQ = 1.25\times 20^2 = 500$
(c) $v = 0 + at = 2.5\times 20 = 50$
Answer: (a) 20 s (b) 500 m (c) 50 m $\mathbf{s^{-1}}$

9 Conditional probability and independence (All boards)

P(B) = 0.55, P(A) = 0.3 and P(A | B) = 0.3.
(a) Find $P(A \cap B)$ and $P(A \cup B)$.
(b) Find P(B | A).
(c) Determine whether A and B are independent.

(a) $P(A \cap B) = P(A \mid B)P(B) = 0.3 \times 0.55 = 0.165$
$P(A \cup B) = 0.3 + 0.55 - 0.165 = 0.685$
(b) $P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{0.165}{0.3} = 0.55$
(c) $P(A)P(B) = 0.3 \times 0.55 = 0.165$, which equals $P(A \cap B) = 0.165$
So A and B are independent
Answer: (a) 0.165, 0.685 (b) 0.55 (c) independent

10 Differentiating sec, cosec, tan and cot (All boards)

$y = \sec(2x)$.

Find $\frac{dy}{dx}$ and its exact value at $x = \frac{\pi}{12}$.

$\frac{dy}{dx} = 2\sec(2x)\tan(2x)$

At $x = \frac{\pi}{12}$ the angle $2x = \frac{\pi}{6}$, so the gradient is $\frac{4}{3}$

Answer: $\mathbf{\frac{dy}{dx} = 2\sec(2x)\tan(2x)}$**;** $\mathbf{\frac{4}{3}}$