Day 271 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Constant acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist passes P with speed 7 m s^{-1} and accelerates uniformly. She travels 141.6 m in the next 12 s, reaching Q. Find (a) her acceleration, (b) her speed at Q, (c) her speed at the midpoint of PQ. Give answers to 3 s.f. if not exact.

(a) $s = ut + \frac{1}{2}at^2$: $141.6 = 7\times12 + \frac{1}{2}\times a\times12^2$, so $a = 0.8$
(b) $v = u + at = 7 + 0.8\times12 = 16.6$
(c) For the midpoint, $s = 70.8$: $v^2 = u^2 + 2as = 7^2 + 2\times0.8\times70.8$
$v^2 = 162.28$
Answer: (a) 0.8 m s^{-2} (b) 16.6 m s^{-1} (c) 12.7 m s^{-1}

2 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily total rainfall (mm) in the same month are:
Perth: mean 25, standard deviation 5.5;
Jacksonville: mean 8.5, standard deviation 5.7.
Make two comparisons.

Means: Perth has the higher mean (25 against 8.5), so on average the total rainfall is greater at Perth
Standard deviations: Jacksonville is larger at Jacksonville (5.7 against 5.5)
so the readings at Jacksonville are more variable and those at Perth are more consistent
Answer: Perth higher on average; Jacksonville more variable

3 Change of sign to locate a root (All boards)

$f(x) = e^x - 5x - 9$.
Show that the equation $f(x) = 0$ has a root α in the interval [3, 4].

f(x) is continuous on the interval
$f(3) = -3.91 < 0$
$f(4) = 25.6 > 0$
A change of sign, so a root lies in [3, 4]
Answer: Sign change: f(3) = −3.91, f(4) = 25.6

4 Parametric equations: Cartesian form (AQA, Edexcel, OCR, MEI, Eduqas)

A curve has parametric equations $x = 1 + 2\cos\theta$, $y = -2 + 2\sin\theta$.
Find its Cartesian equation and describe the curve.

$\cos\theta = \frac{x-1}{2}$, $\sin\theta = \frac{y+2}{2}$

Use $\cos^2\theta + \sin^2\theta = 1$: $(x-1)^2 + (y+2)^2 = 4$
Answer: $(x-1)^2 + (y+2)^2 = 4$, a circle with centre (1, −2), radius 2

5 Equations with sec x and tan x (All boards)

Solve $3\sec^2 x + 9\tan x + 3 = 0$
for $0° \le x \le 180°$. Give answers to 1 d.p.

Use $\sec^2 x \equiv 1 + \tan^2 x$: $3\tan^2 x + 9\tan x + 6 = 0$
$\tan x = -1$ or $\tan x = -2$
$\tan x = -1$: $x = 135.0°$
$\tan x = -2$: $x = 116.6°$
Answer: x = 116.6°, 135.0°

6 Beam on two supports (AQA, Edexcel, OCR, MEI)

A uniform beam AB rests on supports at C and D, with a particle attached at B. Find the reactions at C and D. Take g = 9.8. Give answers to 3 s.f. if not exact.

Moments about C: $R_D\times6.5 = 10\,g\times4.5 + 2\,g\times9.5$
$R_D = 96.5$
Resolving vertically: $R_C + R_D = 12\,g = 117.6$
$R_C = 117.6 - 96.5$
Answer: R_C = 21.1 N, R_D = 96.5 N

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7 Differential equations in context (All boards)

A drink at 75°C is left in a room at 15°C. Its temperature θ °C satisfies $\frac{d\theta}{dt} = -k(\theta - 15)$.

After 10 minutes the temperature is 55°C. Find k, then the time for it to reach 35°C, to 3 s.f.

$\int \frac{1}{\theta - 15}\, d\theta = \int -k\, dt$, so $\ln(\theta - 15) = -kt + C$

t = 0: $\theta - 15 = 60$, so $\theta - 15 = 60e^{-kt}$

t = 10: $40 = 60e^{-10k}$, so $k = \frac{\ln(\frac{3}{2})}{10} = 0.0405$

$20 = 60e^{-kt}$: $t = \frac{\ln(3)}{k} = 27.1$ minutes

Answer: k = 0.0405, t = 27.1 minutes

8 Binomial expansion: (a + bx)^n for rational n (All boards)

Expand $(2 + 3x)^{-2}$ in ascending powers of x, up to and including x^2.
State the range of x for the expansion is valid.

$(2 + 3x)^{-2} = 2^{-2}(1 + \frac{3x}{2})^{-2}$

$2^{-2} = \frac{1}{4}$

Expand $(1 + y)^{-2}$ with $y = \frac{3x}{2}$

Valid for $|\frac{3x}{2}| < 1$, i.e. $|x| < \frac{2}{3}$

Answer: $\frac{1}{4} - \frac{3}{4}x + \frac{27}{16}x^2$; $|x| < \frac{2}{3}$

9 Two-tailed binomial test critical region (AQA, Edexcel, OCR, MEI, Eduqas)

X ~ B(29, p). H_0: p = 0.4 is tested against H_1: $p \neq 0.4$ at the 5% level.

(a) Find the critical region.

(b) Find the actual significance level of the test (3 s.f.).

(a) Each tail ≤ 0.025. $P(X \leq 6) = 0.0233$, $P(X \leq 7) = 0.0570$

$P(X \geq 18) = 0.0135$, $P(X \geq 17) = 0.0329$

Critical region: $X \leq 6$ or $X \geq 18$

(b) Actual level = 0.0233 + 0.0135 = 0.0368

Answer: (a) X ≤ 6 or X ≥ 18 (b) 0.0368

10 Partial fractions and binomial expansion (All boards)

$f(x) = \frac{-6x - 7}{(1 - 2x)(1 + 2x)}$.

(a) Express f(x) in partial fractions.

(b) Hence expand f(x) up to x^2.

(c) State the range of x for which the expansion is valid.

(a) $f(x) = \frac{A}{(1 - 2x)} + \frac{B}{(1 + 2x)}$: A = −5, B = −2

(b) $-5(1 - 2x)^{-1} = -5(1 + 2x + 4x^2 ...)$

Add the series for both fractions.

(c) Need $|2x| < 1$ and $|2x| < 1$

Answer: (a) $\frac{-5}{(1 - 2x)} + \frac{-2}{(1 + 2x)}$ (b) $-7 - 6x - 28x^2$ (c) $|x| < \frac{1}{2}$