Day 270 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Rough incline: find the coefficient of friction (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle slides from rest down a rough plane inclined at α to the horizontal, where $\tan\alpha = \frac{3}{4}$. It moves 17.64 m in 3 s. Find (a) the coefficient of friction, (b) the speed after 3 s. Take g = 9.8.

$s = \frac{1}{2}at^2$: $17.64 = \frac{1}{2}\times a\times 3^2$, so $a = 3.92$
Along the slope: $mg\sin\alpha - \mu R = ma$, with $R = mg\cos\alpha$
$g(\sin\alpha - \mu\cos\alpha) = a$: $9.8(0.6 - 0.8\mu) = 3.92$
$0.6 - 0.8\mu = 0.4$, so $\mu = 0.25$
$v = at = 3.92\times 3$
Answer: $\mu = 0.25$; 11.76 m s^{-1}

2 Exact definite integrals of standard functions (All boards)

Find the exact value of

$$\int_0^{\ln 3} (2e^{2x} + 5e^x)\,dx.$$

Integrate: $[e^{2x} + 5e^x]$
Substitute the limits and subtract: 18
Answer: 18

3 Normal distribution probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X, the height (cm), is normally distributed with mean 175 and standard deviation 7.
Find (a) P(X > 192.5) (b) P(192.5 < X < 199.5), each to 3 s.f.

(a) $z = \frac{192.5 - 175}{7} = 2.50$, $P(Z > 2.50) = 0.00621$

(b) $z_1 = 2.50$, $z_2 = \frac{199.5 - 175}{7} = 3.50$

$P(192.5 < X < 199.5) = \Phi(3.50) - \Phi(2.50) = 0.00598$
Answer: (a) 0.00621 (b) 0.00598

4 Triangle on a coordinate grid (All boards)

The diagram shows triangle ABC.
(a) Find the equation of the line through C perpendicular to AB.
(b) Hence find the coordinates of D, the foot of the perpendicular from C to AB, and the area of triangle ABC.

Gradient AB = 0, so gradient of the perpendicular = undefined (vertical)
Through C: $x = -1$
Line AB: $y - 3 = 0(x - -2)$. Solve to get D = (−1, 3)
Area = ½ × AB × CD = 25
Answer: (a) $x = -1$ (b) D(−1, 3), area 25

5 Pulley, string goes slack (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (7 kg) and B (3 kg) hang on a light string over a smooth pulley, A being 1 m above the ground. Released from rest; A lands without rebounding. Find (a) the tension while both move, (b) the greatest height of B above its start. Take g = 9.8.

A: $7g - T = 7a$; B: $T - 3g = 3a$
$a = 4g \div 10 = 3.92$, so $T = 3(g + a) = 41.16$
When A lands, B has risen 1 m with $v^2 = 2\times 3.92\times 1 = 7.84$
B then moves freely under gravity: extra height $= v^2 \div (2\times 9.8) = 0.4$
Total = 1 + 0.4
Answer: (a) 41.16 N (b) 1.4 m

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6 Recurrence relations: unknown first term (All boards)

A sequence is defined by $u_1 = k$ and $u_{n+1} = 3u_n + 3$.

Given that $u_3 = 57$, find (a) the value of k, (b) $\sum_{r=1}^{4} u_r$.

$u_2 = 3k + 3$, $u_3 = 3(3k + 3) + 3$
$9k + 12 = 57$ so $k = 5$
Terms: 5, 18, 57, 174
Sum = 254
Answer: (a) k = 5 (b) 254

7 Differential equations in context (All boards)

A drink at 102°C is left in a room at 22°C. Its temperature θ °C satisfies $\frac{d\theta}{dt} = -k(\theta - 22)$.

After 15 minutes the temperature is 72°C. Find k, then the time for it to reach 37°C, to 3 s.f.

$\int \frac{1}{\theta - 22}\, d\theta = \int -k\, dt$, so $\ln(\theta - 22) = -kt + C$

$t = 0$: $\theta - 22 = 80$, so $\theta - 22 = 80e^{-kt}$

$t = 15$: $50 = 80e^{-15k}$, so $k = \frac{\ln(\frac{8}{5})}{15} = 0.0313$

$15 = 80e^{-kt}$: $t = \frac{\ln(\frac{16}{3})}{k} = 53.4$ minutes

Answer: k = 0.0313, t = 53.4 minutes

8 Finding an unknown standard deviation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X is normally distributed with mean μ and standard deviation 8.
Given that $P(X > 33) = 0.0668$, find μ to 3 significant figures.

$P(Z > z) = 0.0668$, so $z = 1.50$

$\frac{33 - \mu}{8} = 1.50$

$\mu = 33 - (1.50)(8) = 21.0$

Answer: μ = 21.0

9 Recurrence relations: limits (All boards)

$u_1 = 22$ and $u_{n+1} = -\frac{1}{2} u_n + 18$.

(a) Find the limit L of the sequence as $n \to \infty$.
(b) Find the least n for which $|u_n - L| < 0.1$.

(a) $L = -\frac{1}{2} L + 18$ gives $L = \frac{18}{1 - (-\frac{1}{2})} = 12$

(b) $u_n - L = (u_1 - L)\, r^{n-1} = 10 \times (-\frac{1}{2})^{n-1}$

$|10| \times \frac{1}{2}^{n-1} < \frac{1}{10}$

Check: n = 7 gives 0.15625, n = 8 gives 0.07812
Answer: (a) L = 12 (b) n = 8

10 Double angle equations (All boards)

Solve $\cos 2x - 2\sin x = 1$
for $-180° \le x \le 180°$.

$\cos 2x \equiv 1 - 2\sin^2 x$: $2\sin^2 x + 2\sin x = 0$
Solve the quadratic: $\sin x = 0$ or $\sin x = -1$
$\sin x = 0$: $x = -180°, 0°, 180°$
$\sin x = -1$: $x = -90°$
Answer: x = −180°, −90°, 0°, 180°