Day 27 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily total rainfall (mm) in the same month are:
Heathrow: mean 25.5, standard deviation 1.3;
Leuchars: mean 27.5, standard deviation 5.9.
Make two comparisons.

Means: Leuchars has the higher mean (27.5 against 25.5), so on average the total rainfall is greater at Leuchars
Standard deviations: Leuchars is larger at Leuchars (5.9 against 1.3)
so the readings at Leuchars are more variable and those at Heathrow are more consistent
Answer: Leuchars higher on average; Leuchars more variable

2 Laws of logarithms in terms of p and q (All boards)

Given that $\log_a 2 = p$ and $\log_a 3 = q$,

express $\log_a\left(\frac{\sqrt{2}}{3}\right)$ in terms of p and q.

$\frac{\sqrt{2}}{3} = 2^{\frac{1}{2}} \times 3^{-1}$

$\log_a(2^{\frac{1}{2}} \times 3^{-1}) = \frac{1}{2}\log_a 2 - \log_a 3$

$= \frac{1}{2}p - q$

Answer: $\frac{1}{2}p - q$

3 Arithmetic series: finding n (All boards)

An arithmetic series has first term 15 and common difference 2.
The sum of the first n terms is 392. Find n.

$\frac{n}{2}(2 \times 15 + (n - 1) \times 2) = 392$

$2n^2 + 28n = 784$

$2n^2 + 28n - 784 = 0$

$(n - 14)(2n + 56) = 0$, so $n = 14$ or $n = -28$

n must be a positive integer, so $n = 14$

Answer: 14

4 Product rule: stationary points (All boards)

$y = x\,e^{-4x}$ for $x > 0$.

Find the exact coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = 1 \times e^{-4x} + x \times (-4)e^{-4x}$

$= (1 - 4x)e^{-4x}$

$e^{-4x} > 0$ and $x > 0$, so $\frac{dy}{dx} = 0$ gives $1 - 4x = 0$ and $x = \frac{1}{4}$

$y = \frac{e^{-1}}{4}$; $\frac{d^2y}{dx^2}$ is negative there, so it is a maximum

Answer: $\left(\frac{1}{4}, \frac{e^{-1}}{4}\right)$, maximum

5 Forces in equilibrium with vectors (AQA, Edexcel, OCR, MEI, Eduqas)

Forces $F_1 = (5\mathbf{i} - \mathbf{j})$ N, $F_2 = (9\mathbf{i} + 12\mathbf{j})$ N and $F_3 = (a\mathbf{i} + b\mathbf{j})$ N keep a 5 kg particle in equilibrium. Find (a) a and b. F_2 is then removed; (b) find the magnitude of the acceleration.

(a) Equilibrium: $F_1 + F_2 + F_3 = 0$

$\mathbf{i}$: $5 + 9 + a = 0$, so $a = -14$

$\mathbf{j}$: $-1 + 12 + b = 0$, so $b = -11$

(b) The resultant is now $F_1 + F_3 = -F_2$, of magnitude $\sqrt{(9^2 + 12^2)} = 15$ N

$a = 15 \div 5$

Answer: (a) $a = -14$, $b = -11$ (b) 3 m s^{-2}

6 Surds: equation with surd coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

Solve $x(\sqrt{2} - 1) = -3 - 3\sqrt{2}$, writing x in the form $p + q\sqrt{2}$.

$$x = \frac{-3 - 3\sqrt{2}}{\sqrt{2} - 1}$$

Multiply top and bottom by $\sqrt{2} + 1$.

Numerator: $(-3 - 3\sqrt{2})(\sqrt{2} + 1) = -9 - 6\sqrt{2}$

Denominator: $2 - 1 = 1$

Answer: $x = -9 - 6\sqrt{2}$

7 Connected particles (rough table, string slack) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particle A (10 kg) lies on a rough table, $\mu = 0.1$, joined by a light string over a smooth pulley to particle B (4 kg), 1 m above the floor. Released from rest; B lands without rebounding. Find (a) the acceleration of A before B lands, (b) the further distance A moves. Take $g = 9.8$. Give answers to 3 s.f. if not exact.

B: $4g - T = 4a$; A: $T - 0.1 \times 10g = 10a$

Adding: $(4 - 0.1 \times 10)g = 14a$, so $a = 2.1$

Speed of B and A as B lands: $v^2 = 2 \times 2.1 \times 1 = 4.2$

After B lands, A decelerates at $\mu g = 0.1 \times 9.8 = 0.98$

$0 = v^2 - 2(\mu g)s$, $s = 4.2 \div 1.96$

Answer: (a) 2.1 m s^{-2} (b) 2.14 m

8 Hypothesis test for a Poisson mean (CIE)

Complaints occur as a Poisson process at a mean rate of 3 per day.
After a change, the number recorded in one day is 1.
Test at the 5% level whether the mean rate has decreased.

H_0: $\lambda = 3$, H_1: $\lambda < 3$; $X \sim \text{Po}(3)$

$P(X \leq 1) = 0.199$

$0.199 > 0.05$

Do not reject H_0: insufficient evidence of a change.

Answer: p-value 0.199; do not reject H_0

9 R-form and solving a sin x + b cos x = c (All boards)

(a) Express $12 \sin x - 5 \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give α to 1 d.p.
(b) Solve $12 \sin x - 5 \cos x = 2$ for $0° \leq x < 360°$, giving answers to 1 d.p.

$R^2 = 12^2 + 5^2 = 169$, so $R = 13$; $\tan \alpha = \frac{5}{12}$ gives $\alpha = 22.6°$

$13 \sin(x - 22.6°) = 2$, so $\sin(x - 22.6°) = \frac{2}{13}$

Principal value 8.8°, then use the symmetry of the sine graph

$x = 31.5°, 193.8°$

Answer: (a) $R = 13$, $\alpha = 22.6°$ (b) $x = 31.5°, 193.8°$

10 Discriminant: range of k (All boards)

The equation $x^2 + (2k - 4)x + (k) = 0$ has two distinct real roots.
Find the set of possible values of k.

Discriminant: $b^2 - 4ac = (2k - 4)^2 - 4(k)$

$= 4k^2 - 20k + 16$

Roots of the quadratic in k: k = 1 and k = 4

Required: $b^2 - 4ac > 0$

Answer: k < 1 or k > 4