Day 269 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Smallest whole number solving an exponential inequality (All boards)

£5000 is invested at 6% compound interest per year.
Find the smallest whole number of years for the value to exceed £15000.

$5000 \times 1.06^n > 15000$, so $1.06^n > 3$

$n \ln 1.06 > \ln 3$, so $n > \frac{\ln 3}{\ln 1.06} = 18.85$

$n = 19$

Answer: n = 19 years

2 Indices: solving equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $16^{2x-1} = 8^{2x+3}$.

Write both sides as powers of 2.

$(2^4)^{2x-1} = (2^3)^{2x+3}$

$4(2x - 1) = 3(2x + 3)$

$8x - 4 = 6x + 9$

$2x = 13$

Answer: $x = \frac{13}{2}$

3 Double angle equations (All boards)

Solve $\cos 2\theta - 6\sin\theta = 1$
for $0 \leq \theta \leq 2\pi$.

$\cos 2\theta \equiv 1 - 2\sin^2\theta$: $2\sin^2\theta + 6\sin\theta = 0$

Solve the quadratic: $\sin\theta = 0$ or $\sin\theta = -3$

$\sin\theta = 0$: $\theta = 0, \pi, 2\pi$

$\sin\theta = -3$ is impossible

Answer: $\theta = 0, \pi, 2\pi$

4 Binomial mean and variance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X has a binomial distribution with mean 27 and variance 10.8.
(a) Find n and p.
(b) Find P(X = 27) to 3 significant figures.

(a) $np = 27$ and $npq = 10.8$

$q = \frac{npq}{np} = \frac{10.8}{27} = 0.4$, so $p = 0.6$

$n = \frac{27}{0.6} = 45$

(b) $P(X = 27) = C(45, 27)(0.6)^{27}(0.4)^{18} = 0.121$

Answer: (a) n = 45, p = 0.6 (b) 0.121

5 Parametric equations: points on axes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has parametric equations $x = 2t^2 + 4$, $y = 2t(t^2 - 9)$.
Find the coordinates of the points where the curve meets the x-axis.

$y = 0$: $t = 0$ or $t^2 = 9$, so $t = 0, 3, -3$

$t = 0$: $x = 4$

$t = \pm 3$: $x = 22$

Answer: (4, 0) and (22, 0)

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6 Power, driving force and resistance (CIE)

A 1000 kg car has engine power 45 kW and constant resistance 750 N. Find (a) its greatest steady speed on a horizontal road, (b) its acceleration at 15 m s^{-1} up a hill where $\sin\alpha = \frac{1}{20}$. Take g = 10.

(a) Steady speed: driving force = resistance. P = Fv: 45000 = 750 v
(b) Driving force F = P ÷ v = 45000 ÷ 15 = 3000 N
Up the hill: F − 750 − 1000 g sin α = 1000 a
3000 − 750 − 500 = 1000 a
a = 1.75
Answer: (a) 60 m s^{-1} (b) 1.75 m s^{-2}

7 Partial fractions: improper fraction (All boards)

Express $\frac{x^2 - 6x - 24}{(x+4)(x-4)}$ in the form $1 + \frac{A}{(x+4)} + \frac{B}{(x-4)}$, where A and B are constants to be found.

Divide: $x^2 - 6x - 24 = (x^2 - 16) + (-6x - 8)$
So the fraction is $1 + \frac{-6x - 8}{(x+4)(x-4)}$
$A(x - 4) + B(x + 4) \equiv -6x - 8$
x = −4 gives A = −2; x = 4 gives B = −4
Answer: A = −2, B = −4

8 Implicit differentiation: horizontal tangents (All boards)

A curve has equation $x^2 + xy + 3y^2 = 11$.
Find the coordinates of the points where the tangent is parallel to the x-axis.

$\frac{dy}{dx} = -\frac{2x + y}{x + 6y}$
Horizontal tangent: 2x + y = 0, so y = −2x
Substitute: $11x^2 = 11$, so $x^2 = 1$ and x = ±1
Points: (1, −2) and (−1, 2)
Answer: (1, −2) and (−1, 2)

9 Normal approximation to the binomial (All boards)

X ~ B(100, 0.25). Use a normal approximation to estimate P(X ≥ 24) to 3 significant figures.

np = 25, npq = 18.75, so Y ~ N(25, 18.75)
With continuity correction: P(Y > 23.5)
$z = \frac{23.5 - 25}{4.33} = -0.346$, P(Z > −0.346) = 0.635
Answer: 0.635

10 Particles on two smooth inclined planes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The system is released from rest. Find (a) the magnitude of the acceleration and which particle moves down, (b) the tension. Take g = 9.8. Give answers to 3 s.f. if not exact.

A: weight component along plane = 3 g sin 60°; B: 3 g sin 45°
A moves down, B up (3 sin 60° > 3 sin 45°): F = ma for the system
3 g sin 60° − 3 g sin 45° = (3 + 3)a
(a) a = 0.779
(b) For A: 3 g sin 60° − T = 3 a, so T = 23.1
Answer: (a) 0.779 m s^{-2}, A moves down (b) 23.1 N