Day 268 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A college has 520 students: 160 Arts, 80 Science and 280 Humanities.
A stratified sample of size 26 is taken.
Find the number of students sampled from each group.

Sampling fraction = $\frac{26}{520} = \frac{1}{20}$

Arts: $160 \times \frac{1}{20} = 8$

Science: $80 \times \frac{1}{20} = 4$

Humanities: $280 \times \frac{1}{20} = 14$

Answer: Arts 8, Science 4, Humanities 14

2 Line meets curve: length of chord (All boards)

The line $y = x - 4$ meets the curve $y = x^2 - 2x - 4$ at A and B.
(a) Find the coordinates of A and B.
(b) Find the exact length AB.

$x^2 - 2x - 4 = x - 4$ gives $x^2 - 3x = 0$
$x(x - 3) = 0$: $x = 0$ or 3
A(0, −4), B(3, −1)
$AB^2 = 3^2 + 3^2 = 18$
Answer: A(0, −4), B(3, −1); AB = $3\sqrt{2}$

3 Smooth incline: passing a point twice (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle is projected up a line of greatest slope of a smooth plane inclined at α. It passes A, 68.6 m from the start, after 4 s and again after 5 s. Find (a) the speed of projection, (b) α to 1 d.p. Take g = 9.8.

Deceleration a = g sin α (unknown). $s = ut - \frac{1}{2}at^2$ at both times:
$u\times4 - \frac{1}{2}a\times4^2 = u\times5 - \frac{1}{2}a\times5^2 = 68.6$
Subtracting: $u(5 - 4) = \frac{1}{2}a(25 - 16)$, so $u = \frac{1}{2}a(9)$
Substituting: $\frac{1}{2}a(9)(4) - \frac{1}{2}a\times4^2 = 68.6$, so $\frac{1}{2}a\times20 = 68.6$, $a = 6.86$
(a) $u = 3.43\times9 = 30.87$
(b) $9.8 \sin\alpha = 6.86$, $\sin\alpha = 0.7$, $\alpha = 44.4°$
Answer: (a) 30.87 m s^{-1} (b) 44.4°

4 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily mean visibility (Dm) in the same month are:
Heathrow: mean 29.5, standard deviation 2.1;
Leuchars: mean 21, standard deviation 3.4.
Make two comparisons.

Means: Heathrow has the higher mean (29.5 against 21), so on average the mean visibility is greater at Heathrow
Standard deviations: Leuchars is larger at Leuchars (3.4 against 2.1)
so the readings at Leuchars are more variable and those at Heathrow are more consistent
Answer: Heathrow higher on average; Leuchars more variable

5 Equation after a sequence of transformations (All boards)

The curve with equation $y = -x^2 - 2x + 6$ is transformed by a translation of $\begin{pmatrix}-2\\-2\end{pmatrix}$, followed by a stretch, scale factor 2, parallel to the x-axis.
Find the equation of the image curve.

Translation: replace x by (x + 2) and change y by −2: $y = -x^2 - 6x - 4$

Stretch parallel to x: replace x by $\frac{x}{2}$: $y = -\frac{1}{4}x^2 - 3x - 4$

Answer: $y = -\frac{1}{4}x^2 - 3x - 4$

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6 Partial fractions and integration (All boards)

(a) Express $\frac{7x + 27}{(x + 1)(x + 6)}$ in partial fractions.

(b) Hence find $\int_0^3 \frac{7x + 27}{(x + 1)(x + 6)}\,dx$, giving the answer in the form ln k.

$7x + 27 \equiv A(x + 6) + B(x + 1)$: $x = -6$ gives $A = 4$; $x = -1$ gives $B = 3$

$I = [4 \ln(x + 1) + 3 \ln(x + 6)]_0^3$

$= 4 \ln \frac{4}{1} + 3 \ln \frac{9}{6}$

$= \ln 864$

Answer: (a) $\frac{4}{x + 1} + \frac{3}{x + 6}$; (b) ln 864

7 Geometric series: smallest n (All boards)

A geometric series has first term 6 and common ratio 3.

Find the smallest n for which the sum of the first n terms exceeds 500.

$S_n = \frac{6(3^n - 1)}{(3 - 1)} > 500$

$3^n > \frac{503}{3}$

$n > \frac{\log \frac{503}{3}}{\log 3} = 4.66$

Check: $S_4 = 240.0$, $S_5 = 726.0$

Answer: n = 5

8 Area between curves and the x-axis (with sketch) (All boards)

The diagram shows the curve $y = x(x - 1)(x - 3)$.

Find the total area of the two shaded regions.

$y = x^3 - 4x^2 + 3x$; $\int y\,dx = \frac{1}{4}x^4 - \frac{4}{3}x^3 + \frac{3}{2}x^2$

From 0 to 1: $\frac{5}{12}$ (above the axis)

From 1 to 3: $-\frac{8}{3}$ (below the axis), so the area is $\frac{8}{3}$

Total = $\frac{5}{12} + \frac{8}{3} = \frac{37}{12}$

Answer: $\frac{37}{12}$

9 Parametric differentiation with trigonometry (All boards)

A curve has parametric equations $x = \sec t$, $y = \tan t$.

Find $\frac{dy}{dx}$ in terms of t and the exact equation of the tangent at the point where $t = \frac{\pi}{4}$, in the form $y = mx + c$.

$\frac{dy}{dx} = \frac{dy}{dt \div dx/dt} = \frac{1}{\sin t}$

$t = \frac{\pi}{4}$: gradient $m = \sqrt{2}$, point $(\sqrt{2}, 1)$

$c = y - mx = -1$

Answer: $\frac{dy}{dx} = \frac{1}{\sin t}$; $y = \sqrt{2}x - 1$

10 Car and trailer (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A 1200 kg car tows a 300 kg trailer by a light tow-bar on a horizontal road. Driving force 1450 N; resistances 400 N (car), 150 N (trailer). Find (a) the acceleration and the tow-bar tension. At 8 m s^{-1} the tow-bar breaks. (b) How far does the trailer then travel?

Whole system: $1450 - 400 - 150 = 1500\,a$, so $a = 0.6$

Trailer: $T - 150 = 300\,a$, so $T = 150 + 300\times0.6 = 330$

(b) After the break the trailer decelerates at $150 \div 300 = 0.5$ m s^{-2}

$0 = 8^2 - 2\times0.5\times s$, so $s = 64$

Answer: (a) 0.6 m s^{-2}, 330 N (b) 64 m