Day 266 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Chain rule and gradients (All boards)

$y = \sin^2(3x)$. Find $\frac{dy}{dx}$ and its exact value at $x = \frac{\pi}{9}$.

Let $u = \sin(3x)$: $y = u^2$, so $\frac{dy}{dx} = 2u^1 \times \frac{du}{dx}$

$\frac{du}{dx} = 3\cos(3x)$, giving $\frac{dy}{dx} = 6\sin(3x)\cos(3x)$

At $x = \frac{\pi}{9}$: $\sin(3x) = \frac{\sqrt{3}}{2}$ and $\cos(3x) = \frac{1}{2}$, so gradient = $\frac{3\sqrt{3}}{2}$

Answer: $\frac{dy}{dx} = 6\sin(3x)\cos(3x)$; gradient = $\frac{3\sqrt{3}}{2}$

2 Pulley, string goes slack (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Particles A (12 kg) and B (2 kg) hang on a light string over a smooth pulley, A being 2 m above the ground. Released from rest; A lands without rebounding. Find (a) the tension while both move, (b) the greatest height of B above its start. Take g = 9.8. Give answers to 3 s.f. if not exact.

A: $12g - T = 12a$; B: $T - 2g = 2a$

$a = 10g \div 14 = 7$, so $T = 2(g + a) = 33.6$

When A lands, B has risen 2 m with $v^2 = 2 \times 7 \times 2 = 28$

B then moves freely under gravity: extra height = $v^2 \div (2 \times 9.8) = \frac{10}{7}$

Total = $2 + \frac{10}{7}$

Answer: (a) 33.6 N (b) 3.43 m

3 Sigma notation (All boards)

Evaluate $\sum_{r=6}^{18} (r - 2)$.

Arithmetic series: first term 4, last term 16, 13 terms

$S = \frac{13}{2}(4 + 16) = 130$

Answer: 130

4 Differentiating a^x (All boards)

$y = 5 \times 5^{2x}$.

Find the exact gradient of the curve at $x = 2$ and the equation of the tangent there.

$\frac{dy}{dx} = 5 \times 2 \ln 5 \times 5^{2x} = 10 \ln 5 \times 5^{2x}$

At $x = 2$: $y = 3125$ and gradient = $6250 \ln 5$

Tangent: $y = (6250 \ln 5)x + 3125 - 12500 \ln 5$

Answer: gradient $6250 \ln 5$; $y = (6250 \ln 5)x + 3125 - 12500 \ln 5$

5 Critical values for a binomial distribution (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(40, 0.2)$.

Find the largest value of c such that $P(X \leq c) < 0.01$.

$P(X \leq 2) = 0.00794$

$P(X \leq 3) = 0.0285$

$0.00794 < 0.01$ but $0.0285 > 0.01$

Answer: c = 2

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6 Binomial expansion: (a + bx)^n for rational n (All boards)

Expand $(2 + 3x)^{-1}$ in ascending powers of x, up to and including x^2.
State the range of x for which the expansion is valid.

$(2 + 3x)^{-1} = 2^{-1}(1 + \frac{3x}{2})^{-1}$

$2^{-1} = \frac{1}{2}$

Expand $(1 + y)^{-1}$ with $y = \frac{3x}{2}$

Valid for $|\frac{3x}{2}| < 1$, i.e. $|x| < \frac{2}{3}$

Answer: $\mathbf{\frac{1}{2} - \frac{3}{4}x + \frac{9}{8}x^2;\ |x| < \frac{2}{3}}$

7 Parametric differentiation with trigonometry (All boards)

A curve has parametric equations x = 2cos t, y = sin t.

Find $\frac{dy}{dx}$ in terms of t and the exact equation of the tangent at the point where $t = \frac{\pi}{4}$, in the form y = mx + c.

$\frac{dy}{dx} = \frac{dy}{dt \div dx/dt} = -\frac{1}{2}\cot t$

$t = \frac{\pi}{4}$: gradient $m = -\frac{1}{2}$, point $(\sqrt{2}, \frac{\sqrt{2}}{2})$

$c = y - mx = \sqrt{2}$

Answer: $\mathbf{\frac{dy}{dx} = -\frac{1}{2}\cot t;\ y = -\frac{1}{2}x + \sqrt{2}}$

8 Normal approximation to the Poisson (CIE)

X ~ Po(40). Use a suitable approximation to find P(X ≥ 44), to 3 significant figures.

$\lambda > 15$, so Y ~ N(40, 40) approximately
Continuity correction: P(Y > 43.5)
z = 0.553, so probability = 0.290
Answer: 0.290

9 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that $\sqrt{2}$ is irrational.

Assume $\sqrt{2} = \frac{a}{b}$ with a, b integers with no common factor.

Then $2b^2 = a^2$, so a^2 is a multiple of 2, hence a is a multiple of 2.
Write a = 2c. Then $2b^2 = 4c^2$, so $b^2 = 2c^2$.
So b is also a multiple of 2, contradicting no common factor.
Answer: contradiction, so $\sqrt{2}$ is irrational

10 Ladder in limiting equilibrium (AQA, Edexcel, OCR, MEI)

A uniform ladder AB (20 m, 15 kg) rests with A on rough ground, μ = 0.4, and B against a smooth wall, B being 16 m above the ground. A man of mass 75 kg climbs the ladder. How far along the ladder from A can he climb before it slips?

Foot of ladder is $\sqrt{(20^2 - 16^2)} = 12$ m from the wall
Vertical: R = (15 + 75)g. Limiting friction: F = μR = 0.4(15 + 75)g
Horizontal: S = F (wall reaction S)
Moments about A (man at distance x along ladder, horizontal distance x×12/20):
S×16 = 15 g×6 + 75 g×(12 x ÷ 20)
36×16 = 90 + 45 x (g cancels), so x = 10.8
Answer: 10.8 m