Day 265 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Addition formulae with given sin and cos (All boards)

A is acute and B is obtuse, with $\sin A = \frac{15}{17}$ and $\cos B = -\frac{4}{5}$.

Find the exact values of sin(A+B) and tan(A+B).

$\cos A = \frac{8}{17}$ (A acute), $\sin B = \frac{3}{5}$ (B obtuse), so $\tan A = \frac{15}{8}$, $\tan B = -\frac{3}{4}$

$\sin(A+B) = \sin A \cos B + \cos A \sin B = \frac{15}{17} \times (-\frac{4}{5}) + \frac{8}{17} \times \frac{3}{5} = -\frac{36}{85}$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{15}{8} + (-\frac{3}{4})}{1 - \frac{15}{8} \times (-\frac{3}{4})} = \frac{36}{77}$

Answer: $\sin(A+B) = -\frac{36}{85}$, $\tan(A+B) = \frac{36}{77}$

2 Binomial expansion: a coefficient (All boards)

Find the coefficient of x^4 in the expansion of $(-3 - x)^8$.

Term in x^4: ${}^8C_4 \times (-3)^4 \times (-1x)^4$

$= 70 \times 81 \times 1$

Answer: 5670

3 Smallest whole number solving an exponential inequality (All boards)

Find the smallest positive integer n such that

$(\frac{4}{5})^n < 0.01$.

Take logs: $n \ln \frac{4}{5} < \ln 0.01$

$\ln \frac{4}{5}$ is negative, so dividing reverses the inequality:

$n > \frac{\ln 0.01}{\ln \frac{4}{5}} = 20.64$, so $n = 21$

Answer: n = 21

4 Quadratic from its graph (All boards)

The diagram shows the curve with equation y = f(x), where f is a quadratic.
The turning point is marked, and the curve crosses the y-axis as shown.

Find f(x), giving it in the form $ax^2 + bx + c$.

Turning point $(-3, 8)$: $f(x) = a(x + 3)^2 + 8$

$f(0) = -1$: $a(-3)^2 + 8 = -1$

$a = -1$, then expand

Answer: $f(x) = -x^2 - 6x - 1$

5 Small angle approximations: limits (AQA, Edexcel, OCR, MEI, Eduqas)

For small θ (in radians), use the small angle approximations to find the value that

$\frac{\cos 4\theta - \cos 3\theta}{\theta^2}$

approaches as $\theta \to 0$.

$\cos 4\theta \approx 1 - \frac{16}{2}\theta^2$, $\cos 3\theta \approx 1 - \frac{9}{2}\theta^2$

Numerator $\approx \frac{9 - 16}{2}\theta^2 = \frac{-7}{2}\theta^2$

Divide by θ^2: expression $\approx -\frac{7}{2}$

Answer: $-\frac{7}{2}$

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6 Circle with a given diameter (AQA, Edexcel, OCR, MEI, Eduqas)

A(1, −11) and B(1, 9) are the ends of a diameter of a circle.
(a) Find the equation of the circle, in the form $x^2 + y^2 + ax + by + c = 0$.
(b) Show that C(7, 7) lies on the circle.

Centre = midpoint of AB = (1, −1)
$r^2 = (0)^2 + (-10)^2 = 100$
$(x - 1)^2 + (y + 1)^2 = 100$: $x^2 + y^2 - 2x + 2y - 98 = 0$
(b) $CM^2 = (6)^2 + (8)^2 = 100 = r^2$
Answer: (a) $x^2 + y^2 - 2x + 2y - 98 = 0$

7 Three-set Venn diagram counts (All boards)

Of 44 students, 12 study Maths (M), 22 Physics (P) and 19 Chemistry (C).
8 study M and P, 5 study M and C, 7 study P and C, and 3 study all three.
One student is chosen at random. Given that a student does not study Maths, find the probability that they study Physics.

Venn regions: M only 2, P only 10, C only 10,
MP only 5, MC only 2, PC only 4, all three 3, outside all 8
Not Maths: 44 − 12 = 32
Physics but not Maths: 22 − 8 = 14
Probability = $\frac{14}{32}$
Answer: $\frac{7}{16}$

8 Hinged rod held by a string (AQA, Edexcel, OCR, MEI)

A uniform rod AB (8 kg, 2.5 m) is smoothly hinged at A to a vertical wall, with a 6 kg particle at B. A light string from B to the wall above A holds the rod horizontal, at angle θ to it, $\tan\theta = \frac{12}{5}$. Find (a) the tension, (b) the magnitude of the hinge force. Take g = 9.8. Give answers to 3 s.f. if not exact.

Moments about A: $T\sin\theta \times 2.5 = 8g \times 1.25 + 6g \times 2.5$
$T\sin\theta = (4 + 6)g = 98$, so (a) $T = 106$
Horizontal component of hinge force: $X = T\cos\theta = 40.8$
Vertical: $Y + T\sin\theta = (8 + 6)g$, so $Y = 8g \div 2 = 39.2$
(b) Force = $\sqrt{(X^2 + Y^2)} = \sqrt{(40.8^2 + 39.2^2)}$
Answer: (a) 106 N (b) 56.6 N

9 Venn diagram with algebra (All boards)

The Venn diagram shows the numbers of 37 students in sets A and B.
(a) Find the value of x.
(b) A student in B is chosen at random. Find P(A).

(a) Total: $(x + 3) + (x + 1) + 2x + 5 = 37$
$4x + 9 = 37$, so $x = 7$
(b) Regions: 10, 8, 14, 5
In B: 8 + 14 = 22, of which in A: 8
Answer: (a) x = 7 (b) $\frac{4}{11}$

10 Ladder in limiting equilibrium (AQA, Edexcel, OCR, MEI)

A uniform ladder AB (13 m, 10 kg) rests with A on rough ground, $\mu = 0.3$, and B against a smooth wall, B being 5 m above the ground. A man of mass 50 kg climbs the ladder. How far along the ladder from A can he climb before it slips?

Foot of ladder is $\sqrt{(13^2 - 5^2)} = 12$ m from the wall

Vertical: $R = (10 + 50)g$. Limiting friction: $F = \mu R = 0.3(10 + 50)g$

Horizontal: $S = F$ (wall reaction S)

Moments about A (man at distance x along ladder, horizontal distance $x \times 12/13$):

$S \times 5 = 10\,g \times 6 + 50\,g \times (12\,x \div 13)$

$18 \times 5 = 60 + \frac{600}{13}\,x$ (g cancels), so $x = 0.65$

Answer: 0.65 m