Day 264 **A Level Maths: A Tier 10-a-day: Model Solutions**

1 Double angle equations (All boards)

Solve $\cos 2x + 2\sin x = -3$
for $-180° \leq x \leq 180°$.

$\cos 2x \equiv 1 - 2\sin^2 x$: $2\sin^2 x - 2\sin x - 4 = 0$
Solve the quadratic: $\sin x = -1$ or $\sin x = 2$
$\sin x = -1$: $x = -90°$
$\sin x = 2$ is impossible
Answer: $x = -90°$

2 Indices: solving equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $25^{3x+1} = 5^{3x-3}$.

Write both sides as powers of 5.
$(5^2)^{3x+1} = (5^1)^{3x-3}$
$2(3x + 1) = 3x - 3$
$6x + 2 = 3x - 3$
$3x = -5$
Answer: $x = -\frac{5}{3}$

3 Comparing data sets from the large data set (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Summary statistics for the daily total rainfall (mm) in the same month are:
Perth: mean 14, standard deviation 5.2;
Jacksonville: mean 9, standard deviation 3.9.
Make two comparisons.

Means: Perth has the higher mean (14 against 9), so on average the total rainfall is greater at Perth
Standard deviations: Perth is larger at Perth (5.2 against 3.9)
so the readings at Perth are more variable and those at Jacksonville are more consistent
Answer: Perth higher on average; Perth more variable

4 Beam about to tilt (AQA, Edexcel, OCR, MEI)

A uniform plank AB (mass 15 kg, length 4 m) rests horizontally on supports at C and D, where AC = 0.5 m and DB = 1.5 m. A 6 kg particle is placed at E, AE = x m, and the plank is about to tilt about D. Find x.

About to tilt about D, so the reaction at C is zero and E lies between D and B.
Moments about D: $6\text{ g}\times(x - 2.5) = 15\text{ g}\times(2.5 - 2)$
$6(x - 2.5) = 7.5$
$x = 2.5 + 1.25$
Answer: $x = 3.75$

5 Foot of a perpendicular (All boards)

The line l has equation $2x + y = -6$.
(a) Find the coordinates of the foot of the perpendicular from P$(-6, 3)$ to l.
(b) Hence find the shortest distance from P to l, in exact form.

Gradient of l $= -2$, so perpendicular gradient $= \frac{1}{2}$

Perpendicular through P: $x - 2y = -12$

Solve with l: $x = -\frac{24}{5}$, $y = \frac{18}{5}$

Distance$^2 = (-6 - (-\frac{24}{5}))^2 + (3 - \frac{18}{5})^2 = \frac{9}{5}$

Answer: (a) $(-\frac{24}{5}, \frac{18}{5})$ (b) $\frac{3\sqrt{5}}{5}$

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6 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

For a sample of 7 pairs of values of daily rainfall and ice cream sales, r = 0.9.
The critical value for a one-tailed test at the 1% level is 0.8329.
Test, at this level, whether there is positive correlation between the two variables.

H_0: $\rho = 0$, H_1: $\rho > 0$
Critical value 0.8329
$r = 0.9 > 0.8329$
This is in the critical region, so reject H_0.
There is evidence of positive correlation.
Answer: Reject H_0: evidence of correlation

7 Connected particles on a rough incline (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The system is released from rest and A moves up the rough plane. Find (a) the acceleration, (b) the tension in the light string. Take g = 9.8.

$\sin\alpha = 0.6$, $\cos\alpha = 0.8$; friction $F = \mu R = 0.4 \times 5\, g \cos\alpha$
B: $7g - T = 7a$
A: $T - 5g\sin\alpha - F = 5a$
Adding: $g(7 - 3 - 1.6) = 12a$
(a) $a = 1.96$; (b) $T = 7(g - a)$
Answer: (a) 1.96 m s^{-2} (b) 54.88 N

8 Intersecting or skew lines (CIE (9709); Further Maths on other boards)

l_1: $\mathbf{r} = \begin{pmatrix}5\\5\\2\end{pmatrix} + \lambda\begin{pmatrix}0\\1\\-2\end{pmatrix}$

l_2: $\mathbf{r} = \begin{pmatrix}-1\\8\\2\end{pmatrix} + \mu\begin{pmatrix}3\\-2\\1\end{pmatrix}$

Determine whether l_1 and l_2 intersect. If they do, find the point of intersection.

Equate components: $5 + 0\lambda = -1 + 3\mu$; $5 + 1\lambda = 8 - 2\mu$; $2 - 2\lambda = 2 + 1\mu$
Solving: $\lambda = -1$, $\mu = 2$, and all three equations are satisfied
Point of intersection = (5, 4, 4)
Answer: Intersect at (5, 4, 4)

9 Connected rates of change: cones and bowls (AQA, Edexcel, OCR, MEI, Eduqas)

Water pours into a cone, vertex down, at 10 cm^{3}s^{-1}. The surface radius is always $\frac{h}{3}$.

Find the rate at which the depth is rising when h = 2 cm.

$V = \frac{1}{3}\pi(h/3)^2 h = \frac{\pi h^3}{27}$, so $\frac{dV}{dh} = \frac{\pi h^2}{9}$

$$\frac{dh}{dt} = \frac{dV}{dt \div dV/dh} = \frac{10 \times 9}{\pi h^2}$$

h = 2: $\frac{dh}{dt} = \frac{45}{2\pi}$ cm s^{-1}

Answer: $\frac{45}{2\pi}$ cm/s

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10 Geometric series: smallest n (All boards)

A salary is £40000 in year 1 and increases by 3% each year.
In which year does the salary first exceed £80000?

Salary in year n = $40000 \times 1.03^{n-1}$

$1.03^{n-1} > 2$

$n - 1 > \frac{\log 2}{\log 1.03} = 23.45$

$n = 25$

Answer: year 25